Volga

Volga represents the sensitivity of vega to a change in the volatility. Denoting by P the option price and by σ the (possibly implied) volatility, volga is mathematically given by $\frac{\partial^2 P}{\partial \sigma^2}$; Consequently, Volga stands for:

(a) the first derivative of vega with respect to a change in volatility.

(b) the second derivative of option value with respect to a change in volatility (hence the other name as *gamma vega*).

The volga is also referred to as vomma. For volatility movements, volga is to vega what gamma is to delta (for spot movement). Usually the volatility used is the implied Black Scholes volatility. However, for more complex models, volga can use, for the "role" of volatility, other parameters, very similar in spirit to the implied Black Scholes volatility. One finds a typical example, say, in the case of a stochastic volatility model where the meaning of implied volatility is not straightforward. Two alternative routes to follow are then available, reflecting different views:

- Model parameter sensitivity and risk scenarios: the first approach is to compute the sensitivity of option prices with respect to model parameters and provide this information for risk management purposes. It is then interesting to provide an analysis of the evolution of these model parameters with market instruments scenarios. Typical scenarios can be a parallel shift of the volatility surface, or a local bump of some buckets of implied volatility of liquid options. Other scenarios can be obtained by doing a principal component analysis (or any other factor analysis) on the volatility surface to see the most important historical reshaping of the volatility surface. One can see what the impact of these scenarios on the model parameters is, by recalibrating the model in the new scenario. Using risk rotation, one can then re-translate the risk for these scenarios.

$$Risk = \sum_{j} \frac{\partial \text{Model parameter j}}{\partial \text{Market Instrument}} * \frac{\partial \text{Portfolio}}{\partial \text{Model parameter j}} \quad (1.1)$$

This scenario analysis helps map model parameters (which are in a sense non-economic and theoretical information) to real market instruments and hence to economic information. This helps also trade the model parameters. Moreover, one can see how various market scenarios can be classified (and hence related) in terms of their impact on the model parameters. In a certain sense, the model helps to project in a small dimension space the infinite complexity of the volatility surface. However, one has to be careful as this approach may ignore some orthogonal risk and make various reshapings of the volatility surface look similar simply because their key difference is not captured by the model.

- Bucket volatility market risk: the second approach is to bypass any model parameter sensitivity and watch the impact of market instruments by bumping them, calibrating the model and re-pricing the portfolio. The risk is then displayed for each individual market instrument. Compared to the previous approach, one can also do risk scenario risk by decomposing the scenarios in terms of individual market instruments risk and applying exactly the same analysis.

The interest of volga is to measure the convexity of an option with respect to volatility. An option with high volga benefits from volatility of volatility. Hence, for options with substantial volga, pricing with a stochastic volatility model with high volatility of volatility may change the price dramatically. Commonly used stochastic volatility models are of the type of Heston (1993), or more recently, correlated stochastic volatility with a skew controlled by a CEV process or a shifted lognormal diffusion. In this model, the diffusion of the forward is of the type:

$$dS_t = \sigma_t (S_t + \mu)^\beta dW_t^S \text{ with } d\sigma_t = v\sigma_t dW_t^\sigma \quad (1.2)$$

Volga is easy to compute in the Black Scholes model. For instance, for a call option, volga is given by:

$$\text{Volga} = S_0 e^{-qT} \sqrt{T} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{d_1^2}{2}\right) \frac{d_1 d_2}{\sigma} \quad (1.3),$$

$$\text{where } d_1 = \frac{\ln(S_0 / K) + (r - q)T + \sigma^2 / 2T}{\sigma\sqrt{T}} \quad (1.4),$$

$$\text{and } d_2 = \frac{\ln(S_0 / K) + (r - q)T - \sigma^2 / 2T}{\sigma\sqrt{T}} \quad (1.5).$$

Figure 1 shows the typical shape of volga for a call option.

(In the case of the Heston model, a closed form can also give volga with respect to the spot volatility and one is then led to do a numerical integration of the characteristic function, using the Fourier method).

The figure shows that call options are more sensitive to the volatility of volatility for roughly 25 and 75 delta options[1]. Interestingly, the at-the-money option is very little sensitive to stochastic volatility effect. This can be intuitively understood by the fact that the-at-the money option price P is linear in the volatility σ, as is shown below:

$$P \approx B(0,T) * S_0 \frac{\sigma\sqrt{T}}{\sqrt{2\pi}} \tag{1.6}$$

where T is the option maturity, $B(0,T)$ is the zero-coupon bond paying 1 unit of local currency at time T, and S_0 is the spot value of the underlying.

The concept of volga is integral part of the whole volatility management process. For complex exotics, traders cannot just simply ignore the risk due to the change of vega (see vega). They have to understand the change of the vega with respect to the spot (see vanna) and to the volatility itself.

And to have a better understanding of volga, it is important to stress the variety of models to account for the smile surface. In particular the shape of volga would crucially depend on the choice of model out of the following list of models:

- stochastic volatility models: the correlation effect between the stock and the volatility plays an important role in shaping the vega change.
- local volatitity model where the resulting local volatility is a function of the underlying itself.
- Jumps models with jumps correlated to the volatility parameters.

[1] 25 (more generally X) delta option is common terminology in option trading desks and refer sto an option whose delta is equal to 25% (X%). Because traders are very much interested in delta for hedging purposes, they prefer calling option by the size of their delta to calling option by their strikes.

- Combination of the above, like Lévy processes.
- Discrete type option pricing models that are in fact, discretised versions of the models above. For instance ARCH/GARCH processes are just discrete versions of stochastic volatility models.

Volga is a key concept for barrier options, especially at the barrier level. Volga is also quite fundamental for compound options that are very sensitive to the volatility of the volatility as well as options on volatility (like volatility swaptions) or forward smile options like ratchet or cliquet type options and any mountain range options with cliquet type strike.

Entry category: options

Scope: options

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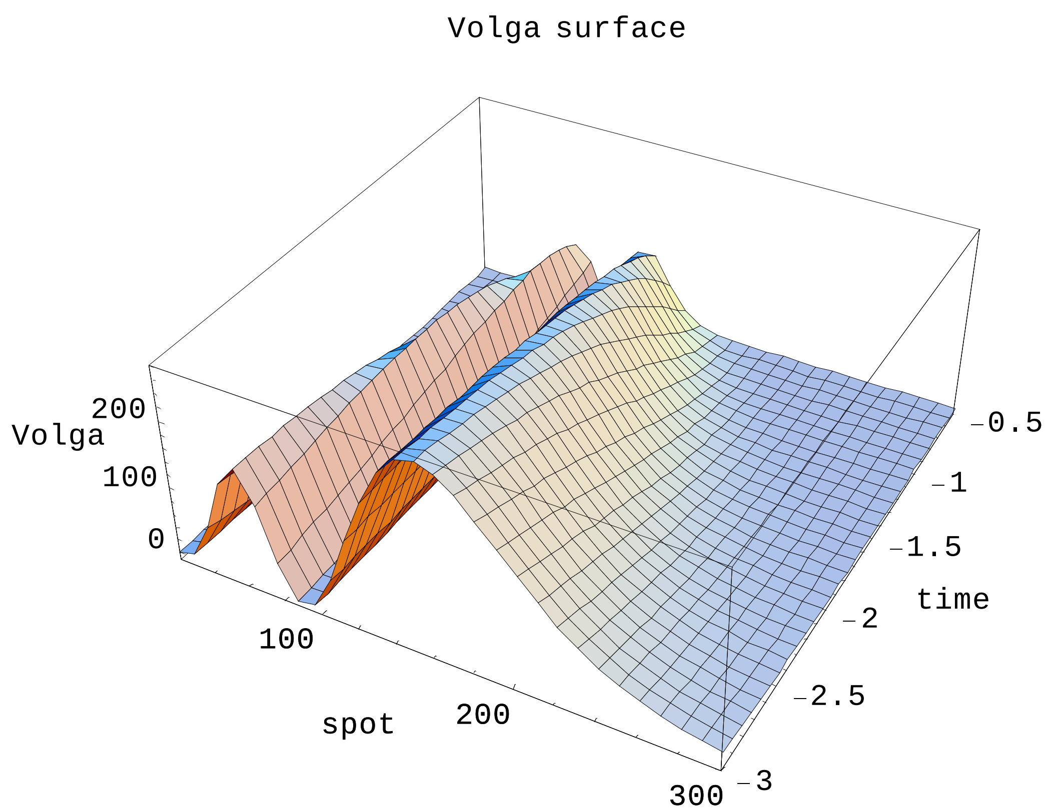

Picture 1: Evolution of the volga surface with time. The parameters are strike 100, a volatility 30%, risk-free rate 5%, and continuous dividend yield 2%. The X axe is the spot varying between 20 and 200 while the Y axe corresponds to the maturity decreasing from 3 year to 0.5.

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