Roll-up/down options

INTRODUCTION

Roll-up and **roll-down** options are options whose strike is automatically reset upwards or downwards while transformed into *up-and-out* or *down-and-out* options. More specifically, in what follows we will assume we are given three strike levels, which we sort in decreasing order, $K_1 > K_2 > K_3$. Then, a **roll-down call** option is a call option with an initial strike of K_1. If during the lifetime of the option, the asset falls below K_2, the option's strike is reset to K_2, and the option becomes a *down-and-out call* with strike K_2 and the barrier set at K_3 (so, if it subsequently goes below K_3, the payoff is zero). Similarly, a **roll-up put** would be initially a put option with a strike K_3. If the asset triggers the level K_2, the option becomes an *up-and-out put* with strike K_2 and the barrier set at K_1. The exotic version of the *roll-up* and *roll-down* options are variations of the roll strategy whereby the initial call is rolled to a call with lower strike a specified number of times before getting transformed into an *up-and-out* or *down-and-out* barrier options.

MOTIVATION AND TARGET MARKET

Roll options allow, as we saw above, to have a strike that resets according to the market movement up to a certain threshold limit. A roll option strategy matches the viewpoint of an investor who thinks that the underlying's price at

maturity should, say, go up, but wants to take advantage of the timing of a temporary shortfall of the market to leverage her position to higher levels with a lower strike call (however, the roll is done by taking additional risk with the barrier type option). More specifically, the investor:

- Buys initially a *call* option
- After a market drop, takes advantage of the trading condition to buy a *call* with lower strike and with a *down-and-out* trigger financed by the sale of the initial call.
- The fact that the initial call has lost most of its value is compensated by the fact that the new option is cheaper than the standard call because of the down-and-out features.

Roll-down and roll-up options took up their name from option traders' jargon: *roll-up* (resp. *roll-down*) an option, according to this jargon, means to close an option position and replace it with a new option position with a higher (resp. lower) strike.

Roll-down or *roll-up* strategies are aggressive strategies suited to small-risk averse investor interested in market timing to leverage their position even more. Like any barrier options, the investor, trader or speculator has to make a trade-off between price reduction and the risk of hitting or not the barrier.

PRICING

We will concentrate on the case of the *roll-down call* (the case of the *roll-up put* can then be treated along the same lines). The *roll-down call* can be seen as *long* two and *short* one *down-and-out call* options (the corresponding

strikes of the three options being pairwise different). The payoff of the *roll-down call* can then be written as the sum of: a) the initial call condition on not triggering the second level K_2 and b) a *down-and-out call* activated if the underlying falls below the threshold K_2:

$$P = (S - K_1)^+ 1_{\{Min_u S_u > K_2\}} + (S - K_2)^+ 1_{\{K_2 > Min_u S_u > K_3\}} \quad (1.1)$$

We then note that the second term in the above expression (: the second option) can be rewritten as the spread between two different *down-and-out call optons* leading to:

$$P = (S - K_1)^+ 1_{\{Min_u S_u > K_2\}} + (S - K_2)^+ 1_{\{Min_u S_u > K_3\}} - (S - K_2)^+ 1_{\{Min_u S_u > K_2\}} \quad (1.2)$$

This shows that the pricing of *roll-down* (or *roll-up*) options share exactly the same problem as barrier options pricing:

- Smile risk at the various barrier levels.
- Liquidity risk at barrier levels.
- Exploding Greeks at barrier levels.

Smile risk

To account for the smile risk, various methodologies can be used with more or less success:

- Use of stochastic volatility models like the one of Hull and White, Heston (1993), Avelaneda and Paras (1995).
- Use of jumps models like the one of Merton (1976) and more generally Levy processes Benhamou (2002).

- Use of local volatility models like the one of Dupire (1993), Derman and Kani
- Combined use of the above models like the one of Andersen, Andreasen (2000), Bates.

Liquidity and exploding Greeks

A way to account for the barrier liquidity risk and exploding Greeks is to shift the barrier level. Another proposed solution is to price with constraints on the Greeks. These constraints are often on the delta or gamma of the hedgin stretegy. For instance, one can compute the cost of the best heding strategy that has a delta of limited size at any times. It can be shown that this approach is similar to shifting the barrier.

Model dependent pricing

Like for any real exotic product, there is no organised market to trade these products. The pricing and risk management of these products is therefore left to the discretion of the bank and strongly depends on its pricing model, hedging strategy and vision of residual risk. Aggressive pricing will depend mostly on the trader's appetite for unhedgeable risk, such as the barrier liquidity risk.

Recent progress in option pricing theory has however shown that barrier options can be statically replicated under the modelling assumption that the underlying follows a local volatility model with jumps. However this theoretical hedge is hard to implement as it involves an infinite set of strikes, which are

not available in the market. In practice, people hedge roll-up puts and roll down like standard options when trading away from the roll-up or roll-down strike. However, as soon as the underlying price moves to the roll level, the options becomes a knock out barrier option and traders need to hedge for the new barrier risk.

Entry category: options

Scope: Target market, pricing and risk management.

Related articles: exotic options and barrier options

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