

## Rainbow options

### INTRODUCTION

A rainbow is an option on a basket that pays in its most common form, a non-equally weighted average of the assets of the basket according to their performance. The number of assets is called the number of colours of the rainbow. At maturity, the assets are sorted by their performance. For instance, a rainbow call with weights 50%, 30%, 20%, with a basket including an equity stock index, a bond index and a foreign currency, pays 50% of the best return (at maturity) between the equity stock index, the bond index and the foreign currency, 30% of the second best and 20% of the third best. One of the common features of rainbow options is to take an average for the underlying, as the Asian version of the rainbow is often cheaper than the vanilla one. Also exotic rainbow are packaged for retail business as structured notes with callable and puttable feature as well as capital guarantee features. A callable rainbow note allows the note issuer to call back the note while a puttable one allows the note buyer to sell it back. Callable puttable versions are becoming also common as they allow flexibility on both sides.

The name of rainbow comes from Rubinstein (1991), who emphasises that this option was based on a combination of various assets like a rainbow is a combination of various colours. More generally, rainbow options are multi-asset options, also referred to as correlation options. Rainbow can take various other forms but the combining idea is to have a payoff that is depending on the asset on the assets sorted by their performance at maturity.

When the rainbow only pays the best (respectively worst) performing asset of the basket, it is also called best-of, (respectively worst-of). Other popular options that can be reformulated as a rainbow option are spread and exchange options. Other variations of the rainbow (given for only the simple case of paying the best performing asset<sup>1</sup>) are described in table 1.

## MOTIVATIONS

Because of the accelerated cross-market integration and greater globalisation of financial markets, traders, investors or speculators have been more and more interested in multi-assets also called correlation products. Moreover, options on many assets are appealing to investor for the following reasons:

- Natural risk diversification. This risk diversification is all the more pronounced that the assets taken for the multi assets products are from very different and unrelated markets. Typical example is to take an equity stock index, a bond index and a currency to have exposure to the equity the bond and the foreign exchange markets.
- Cost efficiency: Multi-asset products are cheaper than the corresponding sum of the individual options. The effect of correlation makes the product competitive.
- Better hedge for portfolio exposure. Correlation products are better hedge for portfolios of investors, traders that includes hundreds of names or for corporate companies that have risks to various market instruments.
- Hedge against correlation. This may seem stupid statement, but correlation risk is very hard to risk manage and correlation products enable

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<sup>1</sup> These options can also be structured to be paying a weighted average of the first best, second best...etc. These options can be call or put and can also be on the first worst, second worst ...etc.

to lock up the implied correlation. Proper delta-hedging enables also to lock up the historical covariance.

Rainbow option offers the additional advantage to provide a weighed average on the best or worst performing assets. The best version supplies attractive returns, while the worst version is often very cheap option.

Rainbow option are either sold to investor as a speculation product or packaged as a structured note for retail market providing attractive multi-asset linked products.

#### PRICING AND RISK MANAGEMENT

Like any complex correlation products, the main challenge for pricing and risk managing rainbow options is the absence of organised markets. A price is therefore determined internally after a close study of the model and the associated risk. Main risks of the rainbows are:

- Correlation risk between the various underlyings: various approaches to be taken: standard and naïve approach of constant correlation between Brownian motion, time dependent correlation and stochastic correlation, cointegration and other statistics tools to capture more robust measure of correlation, copula to control the measure dependence between the assets. Correlation has a positive effect on rainbow overall volatility but negative effect on the forward value of the rainbow. Depending on the maturity and volatility of the underlying assets as well as the weights of the rainbow, the option can be long, flat or short correlation.

- Smile risk: the hedge of the rainbow uses various strikes. This needs to be correctly represented in the modelling of the assets.
  
- Quanto risk for rainbow with asset denominated in other currency than the one of the options. This can dramatically changed the value of the rainbow as the forwards are substantially modified.

Although the academic literature has targeted successfully the case of rainbow option in Black Scholes - work of Stulz (1982) for two assets, generalisation by Johnson (1987) for options with more assets using multidimensional normal cumulative functions, option pricing summary by Rubinstein (1991) – little has been said for model with smile and advanced correlation models. Monte Carlo methods (with American Monte Carlo method to tackle backward looking structures) are the most common way to numerically evaluate rainbow options with a smile model.

Rainbow can be seen as a dynamic basket as the relative weight of each underlyer can change with relative levels. A static hedge with a basket option may perform poorly because of the various switch options (option to switch the weights between the initial ones determined by the forwards and other weights). Another way of looking at it is to see the Rainbow as a basket option with known weights that are effective according to the relative performance of the underlying assets. This can be seen as the product of a basket option time

digital options. The digital options indicate that we are sensitive to the spread volatility between the various underlying assets.

When looking at the rainbow in terms of digital options, it is appropriate to compute the ranking probability of each asset. These ranking probabilities are the probability for each asset to be the most performing, second most performing and so on in order to assess the future composition of the rainbow basket. Part of the risk management tool is then to provide the sensitivity of these probabilities to the various market and model parameters like the forward value of the assets as well as their volatility and correlation.

<b>Name</b>	<b>Payoff</b>
<b>Multi-strike rainbow option</b>	$Max(S_T^1 - K_1, S_T^2 - K_2, \dots, S_T^n - K_n)$
<b>Pyramid rainbow option</b>	$Max( S_T^1 - K_1  +  S_T^2 - K_2  + \dots +  S_T^n - K_n  - K)$
<b>Madonna rainbow option</b>	$Max(\sqrt{(S_T^1 - K_1)^2 + (S_T^2 - K_2)^2 + \dots + (S_T^n - K_n)^2} - K, 0)$

**Table 1:** Variation around the rainbow option

Entry category: options

Scope: correlation products,

Related articles: exotic options, correlation, copulas, multi-asset options.

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<sup>2</sup> The views and opinions expressed herein are the ones of the author's and do not necessarily reflect those of Goldman Sachs