Option position (risk) management

Correct risk management of option position is the core of the derivatives business industry. Option books bear huge amount of risk with substantial leverage in the position. It is therefore crucial for option book runners to have an accurate and efficient risk management system and methodology. If not properly implemented, financial institutions may face similar issues as the ones frequently advertised in the press of distressed financial institutions (like the recent affair of Enron, LTCM, Baring, Worldcom and many more…).

The basics of hedging are quite simple:

- Either part of or all the risk can be transferred to the market in a one-off transaction. The hedging strategy is called a static hedge, as one does not need to rebalance the hedge. In the ideal case where the hedge matches exactly the position, the transaction bears no market risk and is called a back-to-back transaction. This is an ideal case that rarely happens. However it may happen that part of the risk can be decomposed into simple and liquid instrument and can be hedged easily. For instance, this would be for an Asian option, a vanilla European option with half the maturity, for a Bermudan option, the most expensive European option, for a lookback, something between one to two times the European option and so on. The static hedging should be ideally not model dependent but this is rarely the case.

- Or part of or all the risk is dynamically hedged trading regularly, with a frequency that needs to be appropriate for the trade. This is obviously the most common case and we will spend the rest of this article to provide a better understanding of a good dynamic hedging strategy.

Dynamic hedging: the Greeks

In order to assess the risk, one should monitor carefully the option risk ratio, called the Greeks, defined as the price sensitivity with respect to various (model) parameters. More specifically, one calls the option's premium O (or option's price) sensitivity with respect to:

- The spot price S the delta Δ (See *delta*).

$$\Delta = \frac{\partial O}{\partial S} \tag{1.1}$$

  From a trading point of view, the delta represents the amount of underlying to trade to hedge changes of option prices for small movement of the underlying. Logically, an at-the-money option would have a delta close to 50%. Because of the non-linearity of options, the delta can change especially when close to the money. In practice the delta changes quite often and appropriate delta hedging requires frequent rebalancing. The gamma measures precisely the change of the delta with respect to the spot price.

- The implied volatility σ the vega v (*See vega*).

$$v = \frac{\partial O}{\partial \sigma} \tag{1.2}$$

Vega is at its highest for at-the-money option and decreases with shorter maturity. Positions with positive vega would benefit from an increased of implied volatility. If delta hedged, position with positive gamma would also benefit from an increase of the realised volatility. This is because the PNL would depend from the gamma Γ, the realised volatility $\sigma_{realised}$ and the mark-to-market model volatility $\sigma_{\text{mod}el}$ in the following way.

$$dPNL = \frac{1}{2}\Gamma\left(\sigma_{realised}{}^{2}(dW_t)^2 - \sigma_{\text{mod}el}{}^{2}dt\right) \tag{1.3}$$

If the two volatilities are equal, the expected pnl is equal to zero. In the case of a volatility used for the mark-to-market of the option different from the realised one, the expected pnl would be equal to the expectation of half the integral of the gamma times the difference between the two volatilities.

$$E[PNL] = E\left[\int_0^T \frac{1}{2}\Gamma\left(\sigma_{realised}{}^{2} - \sigma_{\text{mod}el}{}^{2}\right)dt\right] \tag{1.4}$$

- The maturity τ the theta θ (*See Theta*).

$$\theta = \frac{\partial O}{\partial \tau} \tag{1.5}$$

Usually positions with positive gamma have negative theta while the opposite is also true and vice versa, since the evolution of a delta hedged portfolio comes only from the theta and the gamma under the assumption of unchanged implied volatility.

$$dPortfolio = \frac{1}{2}\Gamma dS_t{}^{2} + \theta dt \tag{1.5}$$

The theta decreases more rapidly for option close to the maturity. Under Black Scholes

- The interest rates r the rho ρ (*See Rho*).

$$\rho = \frac{\partial O}{\partial r} \tag{1.5}$$

The change of interest rate has two impacts: a positive impact on the forward value of the option's underlying (and hence the intrinsic value) and a negative one on the time value of the option. The impact of the rho increases with the maturity of the option.

Other higher order Greeks includes the gamma, the vanna, the volga, and many more. Computing accurate Greeks is quite difficult as the model assumption and in particular the model distribution can change dramatically their value. A model may provide accurate price while providing erroneous Greeks and hence hedging strategy. A notorious case is the impact of smile models on the dynamics of the delta. Inaccurate computation of Greeks can be quite costly for very singular payoff options such as barrier options, where the Greeks can attain very substantial level. In addition, to provide efficient hedge, one has to take into account shadow Greeks as defined by Greeks that arises from cross effects like delta risk arising from the joint move of the underlying spot and the volatility.

For efficient risk management of option's position, the difference between trending and mean reverting markets is crucial. Although there is never any guarantee that a market is following a trend, if a trader is ready to take a view on this, her optimal delta hedging strategy is not a standard delta. In contrast, she should anticipate the future trend of the market to avoid rebalancing the hedge. Estimation of the optimal hedging strategy is often based on transaction cost models with various risk scenarios. Similarly if a market is mean reverting, it is wise to avoid trading in and out the hedge by taking a more static hedge. Again, the optimal hedging strategy has to be determined with a model examining various risk scenarios, weighted by their risk measure.

Technical, micro and macro indicators can be of certain use to forecast trending and or mean reverting market to apply appropriate market timing strategy.

Last but not least, historical back testing of hedging strategy can help to spot inaccurate risk management.

Entry category: options

Scope: use of Delta, Gamma, Vega, Rho, Theta, simple and complex approaches.

Related articles: Greeks, Delta, Gamma, Vega, Rho, Theta

Eric Benhamou[1]

Swaps Strategy, London, FICC,

Goldman Sachs International

[1] The views and opinions expressed herein are the ones of the author's and do not necessarily reflect those of Goldman Sachs