Vega

Sensitivity of the option price with respect to the volatility, the vega is an important parameter for the risk management of options. Usually computed in the Black Scholes model, it commonly refers to the implied volatility in this model. The vega is sometimes called kappa, thau or zeta. It is a powerful concept that can be extended to volatility curve, matrices and even more advanced volatility type parameters in more complex model

Any financial derivative that has a convex structure will have a vega while the opposite is also true. Consequently, any linear structure like forward has no vega. Mathematically expressed as the first derivative of the option price with respect to the volatility parameter, the vega is often ascertained numerically by repricing the instrument at different levels. In the case of simple model like Black Scholes, the vega is easily derived for most of the vanilla options (call, puts, vanilla Asian, barrier, lookback, simple multi-asset options like rainbow, …). For instance, for a call, the vega is given by:

$$v = \frac{\partial C}{\partial \sigma} = S_0 e^{-qT} \sqrt{T} N'(d_1) \tag{1.1}$$

where $d_1 = \ln\left[\left(S_0 / K + (r-q)T + \frac{\sigma^2}{2}\sqrt{T}\right) / \left(\sigma\sqrt{T}\right)\right]$ (1.2)

with S_0 is the spot value, K is the strike, σ the implied BS volatility, T the maturity of the option, r the risk free rate, q the continuous dividend yield and $N(.)$ is the cumulative density function of a normal distribution.

Usually traders compute a vega corresponding to a discrete move in volatility for a given percentage level. For example, they would compute the change of price of the option for a 1% point move of the volatility.

Except for some pathological cases, like barrier options near the barrier or spread options, vega is for most options positive. Traders are hence long vega, vol or volatility when buying options (like call and put for instance) and short vol when selling options. Positions that are long vega benefit from increase in implied volatility (see implied volatility) but also from actual volatility if the option is being delta hedged.

Like the gamma and theta, the vega follows a bell shape (for simple products). It is at its highest when the option is at the-money and decreases more and more as the strikes diverge. Picture 1 shows the vega for a Black Scholes call for a strike of 100, a volatility of 30%, a risk free rate of 5%, a dividend yield of 2%, with a spot between 20 and 250 and a maturity decreasing from 2 year to the expiry of the option:

In general, most vegas decrease with time except for lookback, reverse knock outs and other barrier option whose vega can increase with time under certain conditions.

Figure 2 shows that the at-the-money vega is very stable to volatility (and hence volatility of volatility in a stochastic model), while it is convex for deep in or out-of-the money vegas. This can be understood easily. For at-the-money option, the option price is linear in the volatility as a simple computation proves it in the Black Scholes model.

$$\text{Call} \quad \text{price} = \sigma\, S_0 e^{-qT} \sqrt{T}\, \frac{1}{\sqrt{2\pi}} \tag{1.2}$$

where σ is the implied BS volatility, S_0 the spot value, T the maturity of the option, q the continuous dividend yield. And more generally, it is easy to check that the second order derivatives of an option price with respect to volatility equals 0 when the strike price equal the forward and is increasingly positive when the strike price is away from it.

To protect a portfolio from changes in the stock price volatility, one needs to make the portfolio vega neutral. In general, a vega neutral portfolio will not be gamma neutral, and vice versa. And it is true that vega and gamma seems to evolve in different ways. However, one can show that vega and gamma are related in simple and homogeneous models (like Black Scholes). They are in these particular case directly proportional (easy to check in Black Scholes).

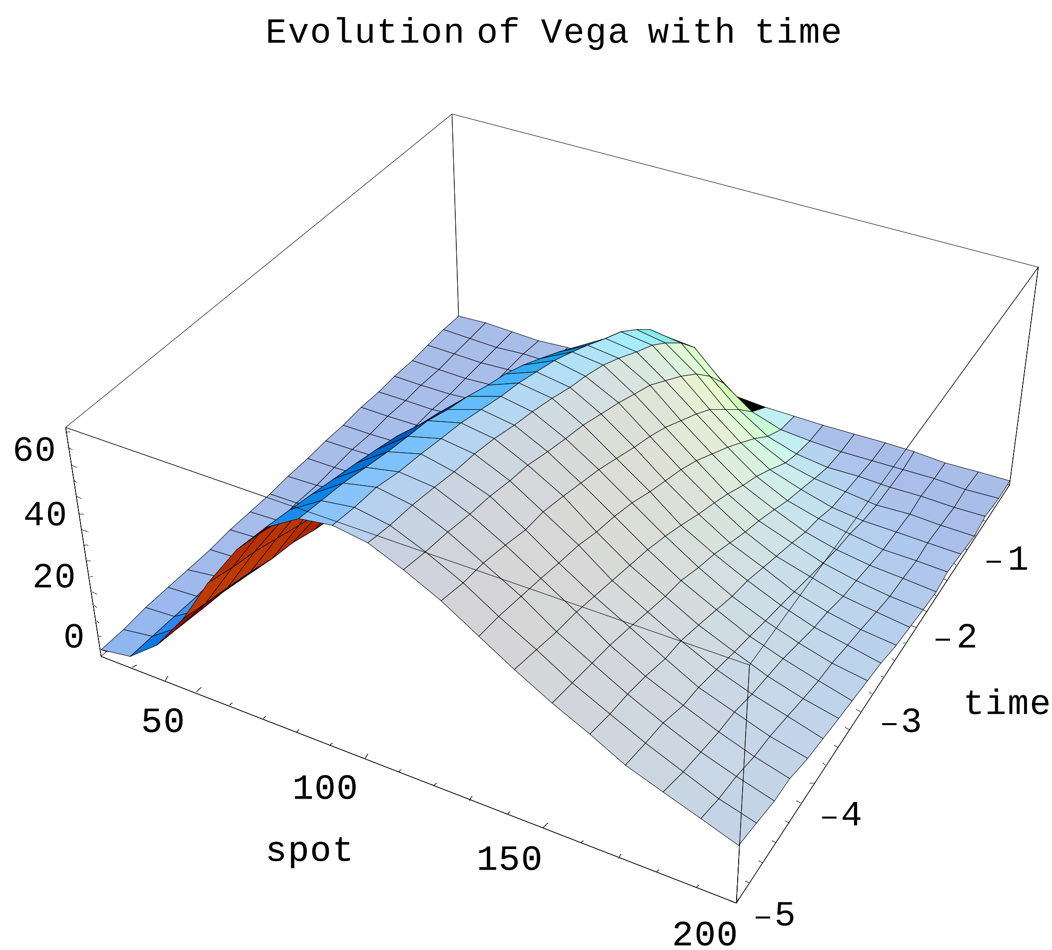
More generally, vega can be shown to be the sensitivity of the expected gamma profits and loss over the life of the option with respect to volatility. Intuitively, the vega pnl resulting from a higher volatility should be equal to the expected gamma profits over the life of the option, should the market becomes more volatile. (see Taleb(1997))

Contrary to the delta and gamma, the vega refers to a model parameter as opposed to a real life move. If the world were Black Scholes, hence with a once and given for ever volatility, the vega would be irrelevant. Vega comes from the imprecision of the model and is sometimes referred to as a model risk as opposed to market risk for greeks like delta or gamma.

For forward starting options, one prefers to use forward volatility sensitivities. These sensitivites are more difficult to estimate as there is no real market for forward volatility. These number may be very dependent of the model used (local volatility models like in Dupire (1993), Derman Kani (1994), or stochastic volatility ones, like in Heston (1992), combination of stochastic volatility and jumps like in Bates (1996), local volatility with jumps, model driven by Lévy processes like in Carr Geman Madan Yor (2000), or Benhamou (2001).

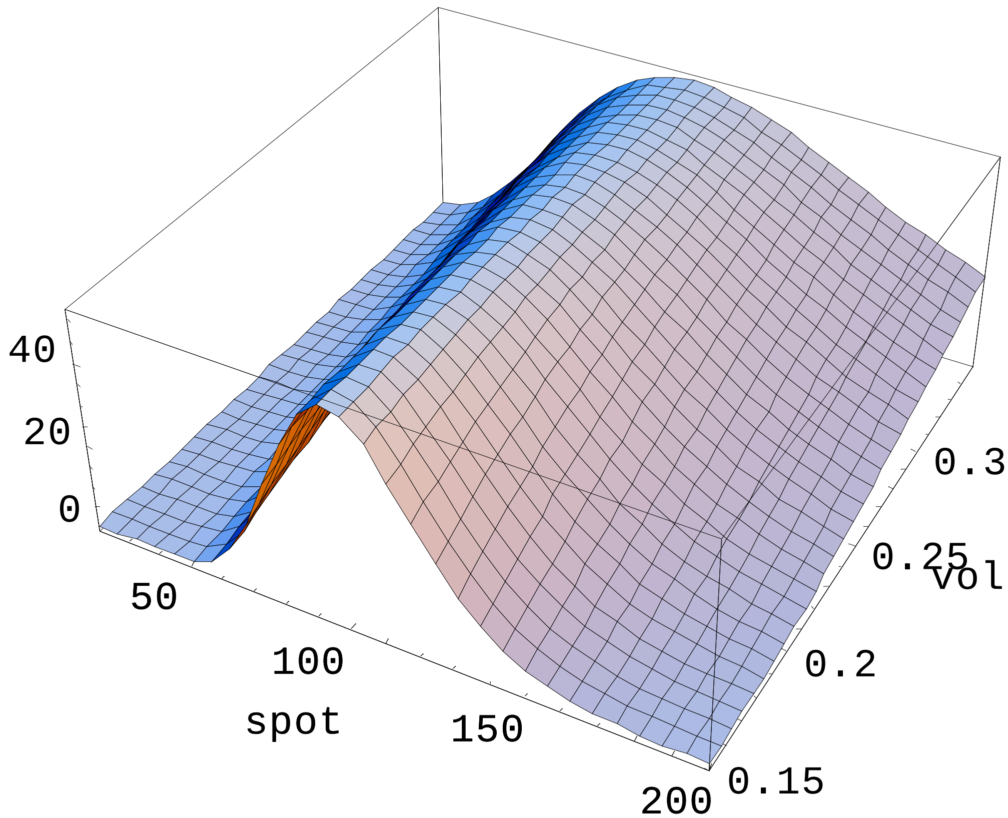
Other generalisation of vega concerns the vega with respect to stochastic volatility model parameters, sensitivity to the local volatility surface, buckets vega. Multi-factors vega refers to advanced methods based on a forward volatility correlation matrix and is used quite commonly for path dependent and deferred start options

Gamma and vega hedging are ways to avoid dynamic hedging. However, since the “greeks” change over time, traders/investors will still need to adjust their hedging positions over time. In the case of barrier options, traders attempting to hedge the risks of vega change with respect to volatility and spot (referred to as the vomma [dVega/dVol] and vanna [dVega/dSpot]), may suffer from the additional entertainment of coping with barrier effects. Good examples of this sort of odd spot effects can often bee seen in the foreign exchange market. A trader who would have written an option on EUR/USD may attempt to defend it when the market is closed to break the level. The market, observing an extremely large bid in front of the level, may then proceed to deluge the trader in sales until the unfortunate trader would throw in the towel. The market would then buy back from him at a lower price as he would unwind the position.



Picture 1: Typical bell shape of the vega of a call option. Note also how the vega decreases with time especially for short expiry when at-the-money. The parameters are strike 100, a volatility 30%, risk free rate 5%, continuous dividend yield 2%. The X axe is the spot varying between 20 and 250 while the Y axe corresponds to the maturity decreasing from 2 year to 0.

Vega for various levels of volatility



Picture 2: Sensitivity of the vega to the volatility (plotted as the second axes). Note how stable the at-the-money vega is with respect to volatility. The parameters are strike 100, risk free rate 5%, continuous dividend yield 2%, maturity 2 year. The X axe is the spot varying between 20 and 250 while the Y axe corresponds to the volatility varying from 15% to 35%.

Eric Benhamou[1]

Swaps Strategy, London, FICC,

Goldman Sachs International

Entry category: options

---

[1] The views and opinions expressed herein are the ones of the author's and do not necessarily reflect those of Goldman Sachs

Scope: options

Related articles: other greeks, gamma, delta, implied volatility, vanna, volga.

References

Bates (1996), Jumps and Stochastic Volatility: Exchange Rate Process Implicit in Deutsche Mark Options, Review of Financial Studies, (9), no 1, p 69-107.

Benhamou (2001), Option Pricing with Levy process, London School working paper, http:\\www.ericbenhamou.fr.st

Carr Geman Madan and Yor (2001), Stochastic Volatility for Lévy Processes, Journal of Computational Finance.

Derman E. and Kani A. (1994), The volatility smile and its implied tree, Goldman Sachs Quantitative Strategy Reports. http:\\www.gs.com

Dupire B. (1993), Pricing and hedging with smiles, Risk Magazine.

Heston (1992), A closed form solution for options with stochastic volatility with applications to bond and currency options, The review of Financial Studies (6), page 327-343.

Taleb N (1997), Dynamic Hedging, John Wiley.