Vanna

Sensitivity of vega (also known as kappa) to a change in the underlying price, the vanna is a second order “cross” Greeks. Like any other cross Greeks, the vanna can be defined in many ways:

(a) first derivative of vega with respect to a change in the underlying.

(b) first derivative of delta with respect to a change in volatility.

(c) second derivative of option value with respect to a joint moved in volatility and underlying.

PRACTICAL USE

Vanna may sound just a nice mathematical concept, but for the big derivatives house, and especially exotic foreign exchange and equity derivatives trading desks, vanna can represent millions of euros or dollars.

Vanna is usually exploited to monitor the vega exposure although some trading desks may also refer to it to monitor their cross gamma risk on delta with respect to volatility.

Exotic options are highly leveraged (see leverage) and need active hedging. Understanding the vega, (change in option value to a 1 percent move in volatility, see vega definition) is crucial for the correct design and maintenance of an effective hedge. As vega moves, the hedging requirements change.

Ignoring it can leave a trader painfully under or over hedged and ultimately nursing substantial losses.

For simple call and put option, it is not too hard to understand the vega. However, when it comes to running a book of exotics derivatives like FX reverse knockouts, equity trigger barrier options and all the panoply of complex exotics like the mountain range equity derivatives, the passport option and Parisian options, it becomes important to monitor the change of vega to various parameters and in particular to the evolution of the spot. This is precisely the point of the vanna. Volga, which is the change of the vega with respect to volatility plays also an important role (see volga).

In the Black Scholes model, the vanna is easy to compute for simple options with closed forms. For instance, this leads to the following closed form in the case of the call option:

$$Vanna = e^{-qT}\sqrt{T}\,\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{d_1{}^2}{2}\right)(1-d_1) \qquad (1.1),$$

where $$d_1 = \frac{\ln(S_0/K)+(r-q)T+\sigma^2/2T}{\sigma\sqrt{T}} \qquad (1.2).$$

But life is not as easy as the simple Black Scholes Merton model. Most common exotics are strongly path dependent. This path dependence is often both in times, meaning that the option depends on volatility points of various maturities, but also in strikes. And often exotics have different and higher exposure to these second-order Greeks than the corresponding vanilla option.

Appropriate risk management needs to account for the correct modelling of the volatility surface referred to as the smile surface (see smile). Using one single vol in a Black Scholes model is just purely wrong for exotic options. This could lead to substantial loss as one would sell options cheaper than most of the street exotic derivatives power houses.

To account for the complexity of the smile and in particular its skew and convexity, one needs to take the route of more realistic models including deterministic local volatility, stochastic volatility and jumps (see smile modelling), and model driven by Lévy processes like the variance gamma process.

Deterministic models assumes that the volatilty is a deterministic function of the spot and time. Variations around the original model of Dupire (1993) and Derman Kani (1993) include a version with jumps (Andersen and Andreasen (1999)):

$$\frac{dS_t}{S_t} = (r_t - q_t)dt + \sigma(t, S_t)dW_t + J_t dN_t \tag{1.3}$$

Stochastic volatility models assume that volatility itself is deterministic, like for instance the Heston model (1993). Variation include the introduction of jumps Bates (1996). Obviously, stochastic models can change considerably the shape of the vanna as they explicitly spedify the correlation between the volaitlity and the spot. If spot and volatility are positively correlated, the holder of the option with positive vanna will benefit from the correlation.

Lévy processes models assume that the stochastic process driving the uncertainty is a more general process, with independent and stationary increments.

On the sell-side of the market, managing the vega risk using vanna can be very tricky as the option may cease to exist brutally. And not surprisingly, single and double knockout barrier options have been heavily advertised by the profession to cause market-makers sleepless night.

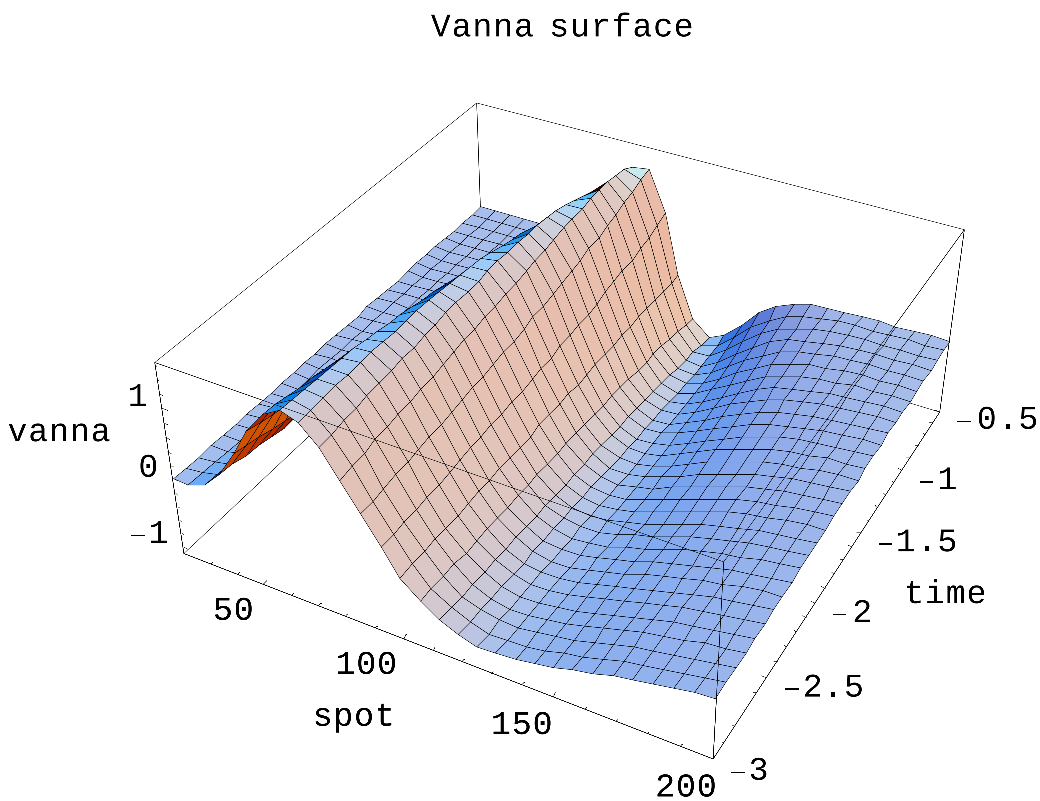
In the simple form, the reverse knock-out structure is a call in the money, with the knockout trigger set above spot. If the market rallies and hits the barrier, the option ends. Compared to a vanilla call, this option is much cheaper as the investor gives up all the call option upside if the spot reaches the barrier. The explosive cocktail of discontinuous payoff function and in-the-money termination makes this product very sensitive to the smile, especially at the barrier level.

Moreover, barrier options can have very different behaviour than the corresponding vanilla option. For instance, barrier option have negative delta and vega when close to the barrier, because of the high chance of knocking out. For similar reasons, the vanna is far higher than the corresponding vanilla in-the-money call.

In addition, exotic dealer may have to cope with jump of the market near important thresholds. This may be due to various reasons, like technical analysis levels or resistances (projection of Fibonnaci numbers, Elliot wave theory, and so forth), but also cluster of barrier options at the same level and subsequent market manipulation by heavy players. Accounting for liquidity holes is also crucial for a good risk management of exotic derivatives. Vanna hedging may be a challenges for large trades All of these makes vega hedging a stressful process.

If there is one market in which volatility risk has to be managed and vanna monitored, it is for sure the foreign exchange exotic derivatives one. The combination of liquidity and very discontinuous and complicated payoffs on a small number of assets have forced foreign exchange dealers to develop sophisticated models to risk manage their number one risk: volatility.

In contrast, the fixed income market has traditionally focussed on the interest rate curve risk, while the equity derivatives market has grown more in the direction of correlation-based, rather than discontinuous-payoff, products.



Picture 1: Evolution of the vanna surface with time. The parameters are strike 100, a volatility 30%, risk free rate 5%, continuous dividend yield 2%. The X axe is the spot varying between 20 and 200 while the Y axe corresponds to the maturity decreasing from 3 year to 0.5.

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[1] The views and opinions expressed herein are the ones of the author's and do not necessarily reflect those of Goldman Sachs

Related articles: vega, volga, greeks, smile, barrier options.

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