Synthetic options

Synthetic options consists in trading a varying position in underlying asset (or futures on the underlying asset[1]) to replicate the payoff profile of a desired option. In practice, traders take position in the underlying asset to replicate the delta (plus some additional terms to capture convexity when this is very significant) of the required option as the delta hedging strategy replicates the option. More generally, a synthetic option position means an option position that is created by dynamic replication.

Synthetic options are used mostly by portfolio manager of large funds. The logic behind is twofold:

- Turn around for liquidity problem in term of cost efficiency and anonymity. Option markets do not always have the level of liquidity to absorb smoothly large option trades. A portfolio manager may not want to use synthetic option to keep anonymity, as a large block transaction would immediately reveal her interest. A portfolio manager may also not be ready to pay a huge liquidity premium for a large trade.
- Turn around for unavailable strike or maturities: the desired strike or maturity may not be offered in the exchange-traded option market.

Synthetic put options have been quite popular by fund managers. And before the 1987 crash, there was massive transaction to synthetise the put options.

[1] One uses often futures to do the synthetic option as the futures market is in most cases more liquid and with much lower transaction costs.

Let us review the case of the put before examining the various risks and drawback implies by synthetic options.

SYNTHETIC PUT POSITIONS

The spot delta Δ of a put is given under Black Scholes (see delta) by

$$\Delta = e^{-qT} N(-d_1) \tag{1.1}$$,

where $d_1 = \frac{\ln(S_0 / K) + (r-q)T}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}$, $N(x)$ is the cumulative normal density function, S_0 is the spot stock price, K the strike price, r the risk free rate, q the continuous yield dividend, T the option maturity and σ the Black Scholes implied volatility.

Creating a synthetic put option implies therefore to sell Δ times the notional of stocks and invest the proceeds into riskless assets. If futures are used, one needs to use the future delta Δ^f equal to[2]:

$$\Delta^f = e^{-qT} e^{-(r-q)T^f} N(-d_1) \tag{1.2}$$,

where T^f is the maturity of the futures contract. When the futures contract is not exactly following the underlying of the option, the portfolio manager can still use futures contract but need to adjust the notional by the β, meaning that the notional should be multiplied by the β.

Example: a fund manager with a portfolio worth $200 million wants to create a synthetic put 1 year, relative strike of 80%. Risk free rate is 5% and dividend yield 2% per annum while volatility is 30%. The spot delta would then be

[2] Ignoring any convexity adjustment between futures and forwards.

$$\Delta = \exp(-2\%*1)*N\left(-\frac{\ln(100/80)+\left(5\%-2\%-30\%^2/2\right)\sqrt{1}}{30\%\sqrt{1}}+0.5*30\%\sqrt{1}\right)$$
$$= -0.1634$$

this would imply to sell 32.6 mio of stock. If the stock declines to 98, the delta becomes -18%, meaning an extra 1.7% need to be sold. Suppose now that the manager decides instead to use futures contract with maturity of 6 month, the future delta is equal to –0.161.

SYNTHETIC OPTION AND REBALANCING

As the stock declines, the delta increases and the portfolio manager sells more and more stock. The opposite is also true. As the stock rises, the delta diminishes and the portfolio manager buys back some of the stocks. Clearly, this is not very optimal as the portfolio manager buys when prices fall and buys when prices rise. With no transaction cost and continuous trading, the strategy is replicating at not cost. But trading is not that easy and one needs clearly to tackle the issue of the optimal rebalancing frequency.

In fact, using transaction cost models like the Leland model, one can get the optimal rebalancing frequency. The other important problems with synthetic option are to cope with:

- Stochastic volatility and non stationary volatility. If volatility moves rapidly the delta computed with a constant volatility can be seriously mispriced leading to erronous dynamic replication .
- Jumps in the market: in the case of a crash like the one of 1987, synthetic replication can not be efficiently performed as the market move to fast to

dynamically replicate the option. On Monday, October 1987, the market moved so fast that portfolio managers with synthetic put option could not sell either stocks or index futures fast enough to protect their position and were hit.

SYNTHETIC OPTION AND STOCK MARKET VOLATILITY

There has been a controversial debate whether synthetic option strategy was increasing or not market volatility. In addition, program trading can follow very similar trading strategy as synthetic option ones. The Brady Commission report on the October 19, 1987, estimated that two thirds of the equity asset were under portfolio insurance using synthetic options. Furthermore, it estimated that on the day of the crash, only one third of the synthetic option position could be exercised, leaving some portfolio manager with substantial losses.

As a matter of fact, following this disastrous trading environment, synthetic option positions became much less popular than before the crash as the asset management industry realised that synthetic option can be quite risky in highly volatility market situations.

Entry category: Options.

Key words: put-call parity.

Related articles: put call parity, traded option markets.

Eric Benhamou[3]

Swaps Strategy, London, FICC,

Goldman Sachs International

[3] The views and opinions expressed herein are the ones of the author's and do not necessarily reflect those of Goldman Sachs

References

Hull, John C, (2000) *Options, Futures, and Other Derivatives*, Fourth Edition, Prentice-Hall.