

Options, pre-Black Scholes

Modern finance seems to believe that the option pricing theory starts with the foundation articles of Black, Scholes (1973) and Merton (1973). This is far from being true. Numerous researchers had worked on building a theory of rational pricing of options and derivatives and a general theory of contingent claims valuation. The way to the final discovery of the Black Scholes model crosses the path of big figures such as Bachelier, the two Nobel prizes Samuelson and Merton Miller.

The idea of using mathematics to predict the future is pretty old and can be retraced back to the seventeenth century with the two French mathematicians, Blaise Pascal and Pierre De Fermat (also famous for the Fermat's Last Theorem). Through various letter exchanges in 1654, the two mathematicians set up the foundation of probability theory. Mathematics cannot predict the future with certainty but can quantify the chance of a given outcome with certainty. Probability theory would set up the background for modern financial mathematics.

The story of options pricing really begins with the French mathematician, Louis Bachelier, who derived a closed formula for the pricing of standard calls and puts in his 1900 PhD thesis dissertation. His assumption, quite revolutionary for his time, is that stock prices follow an arithmetic Brownian motion. In contrast to the standard Black Scholes formula, returns are normal

(as opposed to log normal). He shows that for non dividend-paying stocks, and for zero interest rates, the price of a European call should be:

$$c(S, T) = SN\left(\frac{S - K}{\sigma\sqrt{T}}\right) - KN\left(\frac{S - K}{\sigma\sqrt{T}}\right) - \sigma\sqrt{T}n\left(\frac{S - K}{\sigma\sqrt{T}}\right) \quad (1.1)$$

where S is the stock spot price, K the strike price, σ the (normal) volatility of the stock price (in other words, the instantaneous standard deviation of the stock price), T the time to the option's maturity, $N(x) = \int_{-\infty}^x \exp(-u^2/2)/\sqrt{2\pi} du$

the cumulative normal density function, and $n(u) = \exp(-u^2/2)/\sqrt{2\pi}$ the probability density function of the standard normal distribution. Bachelier, working under the supervision of the famous mathematician Poincarre, was way above his time as the theory of Brownian motion was only pointing his nose. And it took more than sixty years to research to provide new alternative to option pricing theory.

As pointed out by Merton (1973) and Smith (1976), the Bachelier formula ignores any discounting and assumes that stock prices can be negative. This formula makes lots of sense for spread option and any underlying that can be negative but certainly not for stock prices.

Theory of uncertainty and the use of mathematics to financial economics developed in the late fifties with the work of Merton Miller and the Chicago school, producing most of the key ideas of the theory of uncertainty and key discoveries such as the Modigliani Miller (1958) theorem.

However, it is Sprenkle (1961) who the first started to adapt the approach of Bachelier to non-negative prices by assuming lognormal returns. He also assumed that investors were risk averse and come up with a formula of the type:

$$C(S, T) = e^{\rho T} SN(d_1) - (1 - A)KN(d_2) \quad (1.2)$$

with $d_1 = \frac{1}{\sigma\sqrt{T}} \left[\ln\left(\frac{S}{K}\right) + \left(\rho + \frac{1}{2}\sigma^2\right)T \right]$ and $d_2 = d_1 - \sigma\sqrt{T}$, ρ is the average rate of growth of the stock price and A is the degree of risk aversion. This formula, although being very close to the one of Black Scholes did not receive much attention because of the numerous parameters to estimate. One needs in fact to be able to calculate the degree of risk aversion A as well as the average growth of return ρ . Sprenkle (1961) in his article did not give much information how to compute them.

Later, Boness (1964) improved the formula by accounting for the time value of money through the discounting of the terminal stock price, using the expected rate of return of the stock. The formula was changed into

$$C(S, T) = SN(d_1) - Ke^{-\rho T} N(d_2) \quad (1.3)$$

with same definition for d_1 and d_2 as in Sprenkle(1961). Samuelson (1965) allowed the option to have a different level of risk from the stock. He defined α the average rate of growth of the call's value, and came to the following formula:

$$C(S, T) = Se^{(\rho - \alpha)T} N(d_1) - Ke^{-\alpha T} N(d_2) \quad (1.4)$$

with same definition for d_1 and d_2 as in Sprenkle(1961). Samuelson, who was already at that time considered as a very brilliant economists (distinguished by

the Nobel prize in 1970) had already noticed the interest and importance of option pricing theory in economics. This may be why he suggested to one of his young and brilliant students, Merton, to start investigating this in greater details. In another paper, Samuelson and Merton (1969) came with the idea that the option price should be a function of the stock price and that the discount rate used to value the option should be determined by a hedging strategy where investors hold an option and some amount of stocks. They came up with a formula depending on a utility function.

Meanwhile, Thorp and Kassouf (1967) suggested a formula for pricing warrants, which looks similar to the one of Sprenkle (1961).

Although all these approaches provided formulae very close to the one of Black-Scholes, it is only with the groundbreaking work of Black-Scholes and Merton that the option price was explicitly connected to a hedging strategy. The breakthrough of Black-Scholes (1973) was to realise that the expected return of the option price should be the risk-free rate and that by holding a certain amount of stock, now referred to as the delta, the option position could be dynamically completely hedged.

Compared to previous work, the Black-Scholes (1973) formula has the key advantages of giving:

- an explicit hedging strategy for the replication of the call, which only depends on the volatility of the stock price and observable quantities like the risk-free rate, the time to maturity of the option, its strike, the spot stock price.

- a universal price: the option price only depends on the volatility of the stock price and the universal risk free rate. The beauty of Black Scholes is to show that regardless the investor risk aversion, the price of the option should be the same for all investors as they know how to lock in the option value. This was quite different from the previous works that valued the option differently for different risk adverse investors.
- an easy to use formula as the only not easy parameter to estimate is the volatility of the stock price. In fact, at that time, volatility was mainly estimated historically and led to the famous distinction between implied and historical volatility.

It is worth that the Black Scholes formula looks very similar to its predecessor formula:

$$C(S, T) = SN(d_1) - Ke^{-rT} N(d_2) \quad (1.5)$$

with

$$d_1 = \frac{1}{\sigma\sqrt{T}} \left[\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T \right] \quad (1.6)$$

and

$$d_2 = d_1 - \sigma\sqrt{T} \quad (1.7)$$

Like many breakthroughs, it took quite some time to the academic research society to acknowledge the revolutionary of the idea and it is only in 1997 that Black Scholes and Merton received the Nobel price for their key discovery. (see Black Scholes for more details about Black Scholes and Merton).

Eric Benhamou¹

Swaps Strategy, London, FICC,

Goldman Sachs International

Entry category: Options, history of option pricing development prior to BS.

Key words: Samuelson, Bachelier, Black Scholes, Merton

Related articles: Volatility implied, historical volatility, dynamic hedging.

¹ The views and opinions expressed herein are the ones of the author's and do not necessarily reflect those of Goldman Sachs

References

Bachelier L. (1900), Théorie de la Spéculation, Annales de l'Ecole Normale Supérieure, 17

Boness A.J., 1973, Elements of a Theory of Stock Option Value. Journal of Political Economy, 72 (April) 163-75

Black, F. and Scholes, M., (1973), The Pricing of Options and Corporate Liabilities, Journal of Political Economy, Vol. 81, pp. 637-654.

Black, F., (1989), How We Came Up with the Option Formula, The Journal of Portfolio Management, Vol. 15, pp. 4-8.

Merton, R.C., (1969), Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case, Review of Economics and Statistics, Vol. 51, pp. 247-257.

Merton, R.C., (1973), Theory of Rational Option Pricing, Bell Journal of Economics and Management Science, Vol. 4, pp. 637-654.

Modigliani F., Miller M. (1958) The Cost of Capital Corporation Finance and the Theory of Investment. University of Chicago, Chicago IL

Sprenkle C. (1964) Warrant Prices as indicator of expectation, Yale Economic Essays, 1 412-74