

Non standard interest rate caps and floors

Non standard interest rate caps and floors have been developed by marketers and financial engineers to respond to the need of tailor made products to hedge complex interest rates risks. The range of non standard interest rate caps and floors is very diverse and we will focus on the mainstream ones.

Let us remind that a standard cap (respectively floor) is a series of caplets that are single call options on a reference floating money market rate, like Libor 6-month or 3 month. A caplet protects against the rise of the 6-month interest rate. In a standard cap, the Libor resets in advance and are paid in arrears. The notional, on which the interest is paid, is fixed once for all. All the non-standard interest rate cap and floors are variations around the definition of caps. The value of standard caplets is simply the discounted value of the call option on the reference interest rates $L(T_i, T_i, T_{i+1})$ (value at time T_i of the Libor rate whose reference period starts at time T_i ends at time T_{i+1}) times the accrual τ :

$$B(0, T_{i+1}) E^{Q_{T_{i+1}}} \left\{ \tau (L(T_i, T_i, T_{i+1}) - K)^+ \right\} \quad (1.1)$$

where $B(0, T_{i+1})$ is the discount factor at time T_{i+1} , K the caplet strike and

$Q_{T_{i+1}}$ the T_{i+1} forward neutral measure.

- Amortising cap: in this cap, the notional is reduced according to predefined rules. This structure is often sold to investor to replicate their amortising

liability profile. From a trading point of view, amortising caps can be replicated by a basket of caps with different maturities. Accurate pricing of amortising caps implies to determine the volatilities of the corresponding caps.

- Cap in arrears: compared to standard caps, caps in arrears have their rates reset in arrears and also paid in arrears. For common structure, there is a two-business days delay between the reset and the payment, corresponding to the difference between the period end dates and the payment dates. In-arrear caps can be attractive to investors for many reasons: protection against interest rise in an in-arrear swap, interest in delayed payment to take advantage of an expected interest rate rise. In a flat environment curve, in-arrear caps may attract an investor if he believes that rates will rise faster than the yield curve predicts (view of historical rates versus the forwards). In-arrear caps bear a convexity correction risk as the in-arrear caplet value is equal to:

$$B(0, T_{i+1}) E^{Q_{i+1}} \left\{ [1 + \tau L(T_i, T_i, T_{i+1})] * \tau (L(T_i, T_i, T_{i+1}) - K)^+ \right\} \quad (1.2)$$

It is easy to show that in arrear caplets can be priced by static replication as a portfolio of caplets with same maturity but different strikes. Analytical formulation in the Black Scholes model is also easy to derive and shows that the in-arrear caplets does not depend on the correlation between the different Libor as opposed to more exotic products.

- CMS cap/floor: one of the most liquid non-standard caps. In this structure, the rate used is a swap rate with a constant maturity. A CMS cap will for instance be a cap on the 10-year swap rate. CMS cap have been widely

used by insurance companies to solve their solvency problem (protect themselves against the rise of long dated interest rates). CMS caps are tailored instruments to hedge long dated interest rates positions. CMS caps looks attractive in a flat yield curve environment to an investor who thinks that rates will rise faster than the yield curve predicts. Like in arrear caps, the option bear a convexity risk arising from the mismatch between the reference rate and the compounding period used. The value of the CMS caplets is given by :

$$B(0, T_{i+1})E^{Q_{T_{i+1}}} \left\{ \tau(S(T_i, T_i, \dots, T_{i+n}) - K)^+ \right\} \quad (1.3)$$

where $S(T_i, T_i, \dots, T_{i+n})$ denotes the value at time T_i of the constant maturity swap rate with swap dates (T_i, \dots, T_{i+n}) . Similarly to cap, a static replication with a portfolio of swaptions can be achieved. Other pricing includes Monte Carlo and approximation methods.

- Barrier cap also known as trigger cap, knock-in/out cap are caps that are activated or terminated if a specific reference rate has triggered a certain level. Knock-out caps are very attractive to investors that think that the level will not be triggered. Barrier caps are much cheaper than the corresponding vanilla cap as the investors gives the upside to be knocked-out (in a knock-out cap) or not to be ever knocked-in (in the knock in case). Standard trigger caps have a fixed barrier level, monitored continually during the lifetime of the caps. There are many exotic barrier caps: discrete barriers options (barrier level only monitored discretely at certain dates), window barrier (barrier active only during a certain

timeframe), barrier level changing value at fixed points (step up barrier options). The barrier level may also need to be breached for a certain number of days (Parisian barrier options), or may be triggered depending upon how far the triggering underlying asset has gone from the barrier level (soft barrier). There can also be two barrier level (double barrier also referred to as corridor options). Often the reference rate used to trigger the underlying asset is a different rate than the one used to compute the caplet payoff. The barrier option is then referred to outside barrier option. Typical structure is CMS cap with a barrier on the 6 month-Libor. Proper hedging of barrier options close to the barrier level is more an art than a science. Indeed, barrier caps show strong discontinuities in their Greeks closed to the barrier and are often hedged by call spread positions.

- Pay-as you go cap (also known as instalment cap): in a pay as you go cap, the investor pays a small initial premium compared to the corresponding vanilla cap and decides shortly before each reset whether to pay and keep the cap or not. The holder of the periodic cap can cancel the cap when she thinks she does not need it any more. A pay-as-you go is a cheap solution to investors that think that interest rates will eventually stabilise below the strike level or will spike shortly before decreasing below the capped level. Pay-as-you go from a trading point of view resume to compound option and bear an important risk in term of volatility of volatility.
- Asian cap (also know as average cap): in this structure, the rate used is an average of the reference rates. The strike can be a fixing of the reference

rate (floating Asian cap) or fixed (fixed Asian cap). The average may be computed in a number of ways: arithmetic, geometric, weighted, partial averaging over a window. In general, Asian caps are cheaper than the corresponding cap as the averaging tends to offset high values with low values. Asian caps are widely used, as they are cheap and less sensitive to brutal market condition variations and or manipulations.

- Lookback cap: cap on the maximum or minimum of a reference rate. Like Asian structure, the strike can be the reference rate (flexible lookback) or a fixed level (fixed lookback). Lookback caps are quite expensive structures and used for special occasions.
- Quanto cap/floor. Compared to a standard cap, a quanto cap is paid in another currency than the one in which the reference rate is denominated. Quanto structures are particularly popular to be able to take advantage of the funding advantage of a foreign market without taking any foreign exchange risk.
- Ratchet cap (cliquet) or sticky cap, step up cap: in all these structure, the previous level of the reference rate determines the strike of the different caplets. In a step up cap, the strike increases as the reference rate increases. Ratchet caps are the interest rate derivatives equivalent of equity cliquets and bear similarly an important forward smile risk.

- Flexible caps, chooser non-chooser cap, auto cap: in this structure. Quite popular, autocaps (also referred to as non-chooser cap) are a cheap alternative to vanilla cap as only the n th first caplets out of m th ($n < m$) are exercised. In a chooser cap (also referred to as chooser cap), the investor has the right to choose the n th caplets to exercise. Chooser and non-chooser caps bear an important correlation risk.
- Power cap. Pays a call on the reference rate to a given power α . Accurate pricing can be done via static replication.
- Exotic caps like shout caps: in a shout cap, the holder has the right at any point in time during the lifetime of the option to lock in the minimum value of the option by shouting that the option payoff should be based on the reference rate value observed at that time.

Entry category: options

Scope: ratchet caps (cliquet), sticky caps, flexi caps, auto cap, chooser caps, step up caps.

Related articles: interest rate caps and floors, exotic options.

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