lambda (options leverage)

INTRODUCTION

The lambda (option leverage) is defined as the percentage change in an option price divided by the percentage change in an underlying price. Also called effective gearing or also leverage factor, the lambda is a measurement of the option's and defined mathematically by the following ratio:

$$\lambda = \frac{\partial C / C}{\partial S / S} = \frac{S}{C}\frac{\partial C}{\partial S} = \frac{\partial \ln C}{\partial \ln S} \qquad (1.1)$$

Although some authors called also lambda what is commonly called as the vega and defined as the derivatives sensitivity with respect to the volatility, the usage of lambda to mean option leverage have become predominant in option pricing literature.

From the mathematical definition (1.1), one can see that the lambda is:

- a by-product of the delta since it is simply the product of the delta time the ratio of the underlying price over the option price[1]
- equal to the logarithmic delta or in other words equal to the logarithmic derivative of the option price with respect to the underlying price

Because of its straightforward relationship with the delta and its non-direct relationship with hedging, the lambda is not a very widely used Greek (see Greek). Traders prefer to use the delta as this provides them their risk profile

directly. Lambda can be a useful measure for leverage indication when selling option to clients.

Typical value of the lambda for standard option is around 5 to 15 (for instance with 20% volatility and a maturity of 2 years, the lambda of the at-the-money option is around 8.5). In contrast to the delta, the lambda is only convex and less sensitive to the option maturity although varying quite substantially for deep out-of-the-money option (see figure 1 and 2 for a comparison of the evolution of the lambda and the delta with respect to time, and see figure 3 and 4 for the same comparison but with respect to volatility).

LEVERAGE AND RISK

As we can see in the different figure 1 and 3, options are highly leveraged. For a very deep out-of-the money call, leverage can be as high as 20 to 40 times. In addition, this leverage can be even more pronounced for exotic derivatives that have embedded leverage like power options, leveraged note and various type of barrier options (see leverage options for a discussion on power derivatives). Like everything in finance, there is no free lunch. More leverage means more potential upside but also much more risk. A leverage of 20 implies that one has to trade (in notional terms) 20 times the premium of the option to make the same gain or loss using the underlying. It is therefore wise to assess fairly the high level of risk embedded in option before trading them.

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[1] The ratio of the underlying price over the option price is often referred to as the gearing or leverage and should not be confused with the effective leverage which is in fact equal to the gearing times the delta.

The history is full of scandals of companies and people burning themselves while trading option inappropriately or taking too risky positions.

- Orange County loss of $1.6 billion in 1994. The orange county example is probably the first big loss on derivatives in the modern day option trading. The Orange county treasurer Robert Citron in the early 90's invested part of the Orange County fund assets into leveraged inverse floater notes (see reverse floater note) paying a coupon of the type

$$Max\left(FixedRate - \beta 6mUSLibor, 0\right) \tag{1.2}$$

  with β significantly higher than 1 providing a substantial leverage compared to standard option.

- Proctor and Gamble: loss of $200 million. P&G bought from Bankers Trust a $200 million structured swap where P&G receives fixed and pays for the first six months a fixed rate and after a structure coupon of the type

$$FloatingRate - 75bps + 1\% * Max\left(\frac{98.5}{5.78}Y_5 - P_{30}, 0\right) \tag{1.2}$$

  with Y_5 the five years treasury yield and P_{30} the price of the 30-year Treasury bond. Obviously this option is one sided[2] as when treasury yield rises, the price of the Treasury bond falls hence the option becomes more and more in the money.

- Baring total loss of $1.3 billion: a relatively simple story of a rogue trader Nick Lesson selling straddles on the Nikkei, and buying Nikkei futures to increase even his risk. The Nick Lesson trades stood in opposite direction with delta hedging strategy as he was buying Futures with an option position requiring to short them.

- Long Term Capital Management: loss of $3 billion due to various highly leveraged positions. We will devote the next paragraph as this is a very interesting story.

LONG TERM CAPITAL MANAGEMENT LEVERAGE STRATEGY

The leverage taken by Long Term Capital Management (LTCM) has been phenomenal. LTCM was a highly admired hedge funds founded by very well known and talented people like John Meriwether, well known bond trader at Salomon, Myron Scholes and Robert Merton, both Nobel laureates. It managed to get very interesting deals with other counterparties and in particular to leverage tremendously their positions:

- Leverage $4.8 billion of asset into $100 billion positions, with a total position in the swap market at one time over $1.25 billion representing 5% of the market
- Low collateral or no collateral requirement on many deals.

LTCM took many leverage bets on the relative value of certain market instruments out of which the costly notorious ones are:

- EMU convergence trade using European government bonds: short the overpriced German government bonds and long the cheaper other countries' bonds. Although this trade finally became successful, it originally created huge losses in 1998.

[2] Ignoring at first sight the exposure to the slope of the curve.

- Tightening of the spread between Brazil and US Treasuries. Long Brazilian bond and short the US Treasury bonds. In contrast, the spreads widened.
- Flattening of the German interest rate curve: short the 10-year bond and long the 30-year bond. However the curve steepened even further
- Volatility spread between short and long terms volatility using swaption straddles: long long-dated swaption straddles and short short-dated swaption straddles. However the short dated volatility surged because of the Russian crisis

All these positions were done using option to leverage the capital used and led to a spectacular loss for the hedge funds. Estimate of their lambda, although very difficult to assess was around 20 times, with certain positions with a lambda over 40 times.

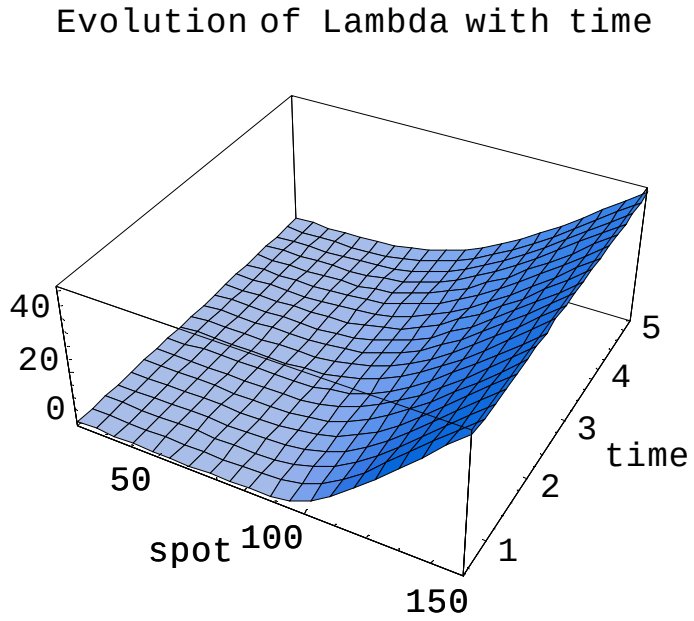

Figure 1: Shape of the lambda of a European call for a time varying from 1 month to 3 year and for a spot between 20 and 150.The strike is taken to be 100, the volatility is 20%, the risk free rate is 5% while the continuous dividend yield is 2%

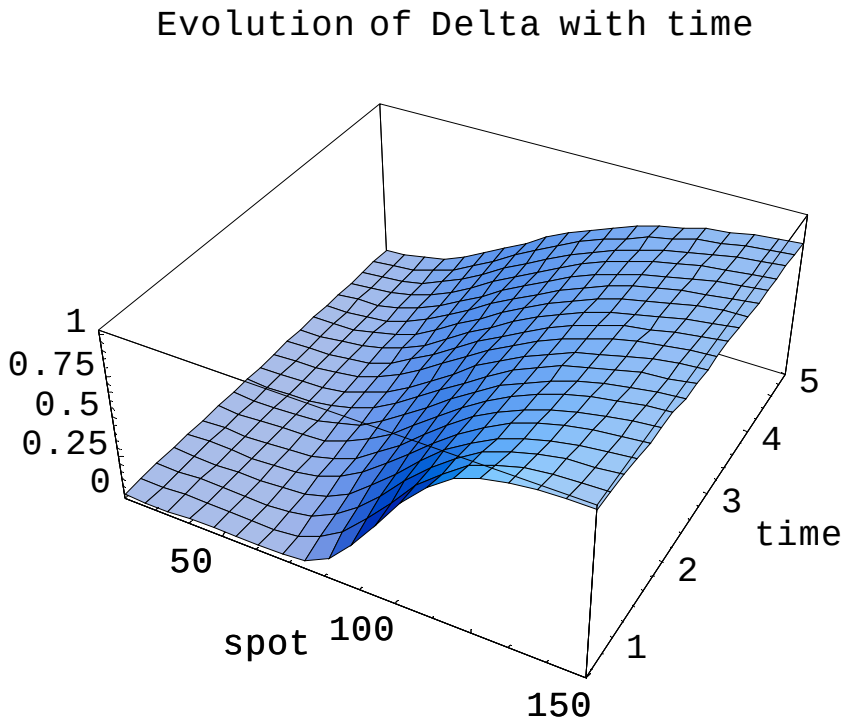

Figure 2: Shape of the delta of a European call for a time varying from 1 month to 3 year and for a spot between 20 and 150.The strike is taken to be 100, the volatility is 20%, the risk free rate is 5% while the continuous dividend yield is 2%

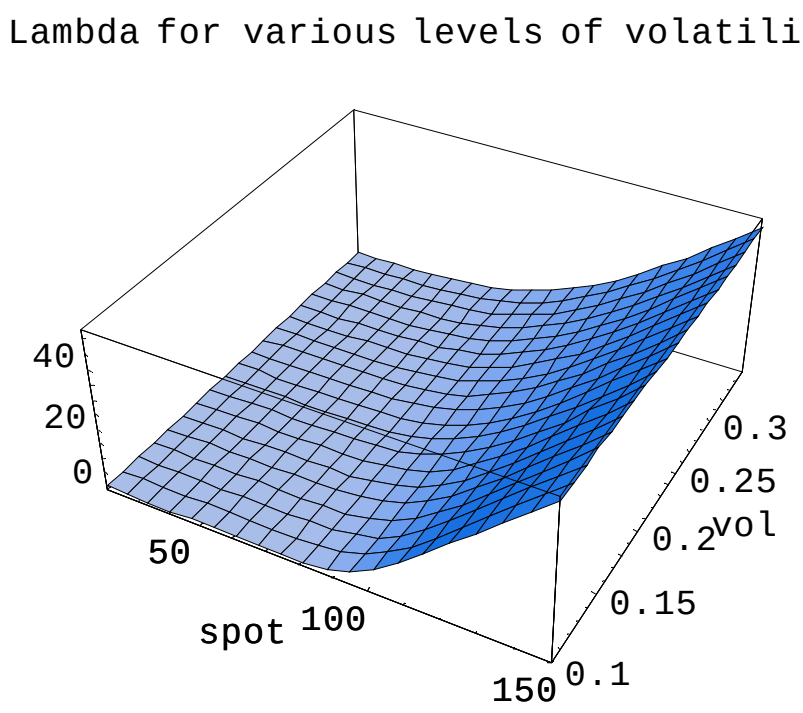

Figure 3: Shape of the lambda of a European call for a volatility varying from 10% to 35% and for a spot between 20 and 150.The strike is taken to be 100, the maturity is 2 years, the risk free rate is 5% while the continuous dividend yield is 2%.

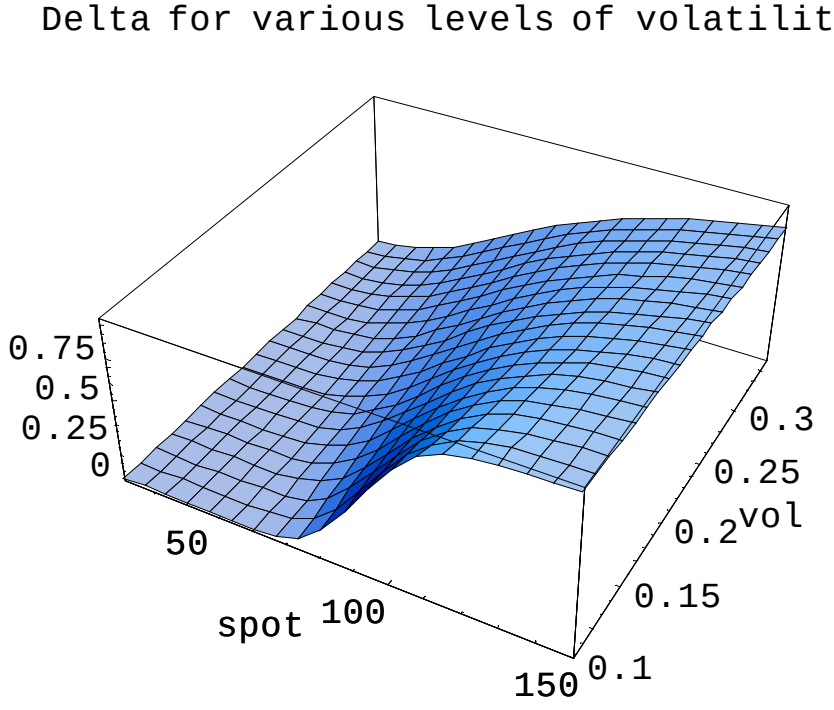

Figure 4: Shape of the delta of a European call for a volatility varying from 10% to 35% and for a spot between 20 and 150.The strike is taken to be 100, the maturity is 2 years, the risk free rate is 5% while the continuous dividend yield is 2%

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Related articles: the Greeks, leverage factor.

[3] The views and opinions expressed herein are the ones of the author's and do not necessarily reflect those of Goldman Sachs

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