Immunisation

Immunisation refers to a hedging strategy in asset-liability management consisting in offsetting the risk of some assets or liabilities by an opposite position in other assets with very similar risks profile. Immunisation is a concept mainly developed for bond portfolios although it can be extended to other assets. In the case of partial hedging of the risk, one calls this strategy partial immunisation. In contrast, absolute immunisation for bond portfolios refers to the hedging with a bond with same duration and convexity. The concept of duration (and also convexity) plays a fundamental role in immunisation theory, hence the other name for immunisation as duration matching. In theory, a perfect immunisation is only attained if the asset to be hedged and the underlying used are perfect substitute.

The usual way of quantifying interest risks for a bond portfolio is to use the duration and convexity. These two concepts have a long history in asset-liability management, dating back to Macaulay (1938).

Duration (see duration) measures the sensitivity of a bond portfolio's value to changes in yield. In its simple form, referred to as the Macaulay duration, it is mathematically given by minus the logarithmic derivative function of the bond price with respect to its yield.

Suppose that a bond pays a stream of coupon c_i at time t_i, with the final coupon c_n including the notional of the bond and y is its continuously compounded yield. The bond price B, is then given by the sum of the discounted cash flows:

$$B = \sum_{i=1}^{n} c_i e^{-yt_i} \qquad (1.1)$$

The duration is defined as:

$$D = \frac{\sum_{i=1}^{n} t_i c_i e^{-yt_i}}{B} = -\frac{1}{B}\frac{\partial D}{\partial y} \qquad (1.2),$$

Which can be written as a weighted sum of the different payment times:

$$D = \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-yt_i}}{B} \right] \qquad (1.3)$$

The term in square brackets is the ration of the present value of the payment at time t_i over the value of the bond, that is the sum of the present value of the payments to come.

Duration provides that the risk of the bond (in terms of change of the yield) is given at first order by:

$$\frac{\Delta B}{\Delta y} = -BD \qquad (1.3)$$

It relies on the parallel shift of the yield curve (see duration for a discussion of more advanced duration concept). Apart from obvious hedging purposes, duration also allows to compare investments with different horizons in a simple way. Suppose that a fixed coupon bearing bond, with a maturity of 10 years has a duration of 7 and a half year. A zero coupon bond maturity 10 years has obvious 10 year duration. Duration explains that the investment in the zero coupon bond contains more risks (to the change of the yield) than the fixed coupon bearing bond.

Convexity gives a finer granularity to the interest rate risk analysis by examining second order risk. Mathematically, convexity is defined as the second derivatives of the bond price with respect to the yield:

$$c = \frac{\partial^2 B}{\partial y^2} \tag{1.4}$$

The fundation idea of immunization theory is then to match both the convexity and the duration of the assets with the liabilities, using specific bonds with liquidity.

In modern financial terms, immunisation (or dedication) theory is similar to a semi-static hedge for a bond portfolio. It is a passive strategy that guarantees that the bond portfolio bares hardly any risk. Occasionally the portfolio manager may need to re-hedge as it is only a second order hedge. Higher moments may on the long term affect the portfolio. As time passes, the duration and convexity of the portfolio and the liabilities may shift. The portfolio must check periodically the quality of the immunisation

For high yield bond, immunisation plays an important role, as interest risk can be quite substantial. Immunisation ignores at first sight any credit valuation of counterparty. In order to gauge the vulnerability of their portfolios to interest rate risk, fund managers may do various scenario analyses and stress testing.

This preliminary analysis would help them structure appropriate immunisation strategies. One needs also to model accurately any embedded option in the bond as the bond's duration and convexity can be considerable modified by the option component.

Entry category: mathematical models

Related articles: yield curve modelling.

Eric Benhamou[1]

Swaps Strategy, London, FICC,

Goldman Sachs International

[1] The views and opinions expressed herein are the ones of the author's and do not necessarily reflect those of Goldman Sachs

References

Fabozzi and Pollack, Handbook of Fixed Income Securities