Fixed income markets (overview)

Fixed income markets encompass all interest rate financial instruments, like a bond, money market instrument, swaps, caps floors, swaptions and more generally any interest rates derivatives. Investment banks often split their trading activity between equity, foreign exchange, fixed income, commodity markets as they are all very different markets.

1. MARKETS AND INSTRUMENTS

The original motivation for the fixed income market was to enable the borrowing and lending of money via debt financing with instruments like bonds, Futures and FRAs (Forward Rate Agreement). However, many other instruments have been developed to allow flexible hedging of debt as well as investment and speculation in these markets: swaps, but also options, like caps, floors, swaptions and various more exotic instruments. Usually, one divides the fixed income markets into:

- Simple instruments, mainly swaps and bonds
- Exotic interest rate derivatives, such as exotic caps, and swaptions
- Credit derivatives as a by-product of fixed income markets, as the credit protection concerns directly bonds and debt instruments.

SIMPLE INTEREST RATE INSTRUMENTS

The simple instruments[1] also referred to as the vanilla products of the fixed income markets are generally liquid instruments, well known by the various fixed income market participants and whose pricing is relatively easy. The list of such vanilla instruments includes:

- Money market, which provides the rate at which banks can lend or borrow money, plus or minus a spread depending on their credit rating.
- Bonds: these can be either government, agency and sovereign bonds, or corporate bonds. Bonds can have various exotic features that makes them not as vanilla as the school case bonds[2]. The (liquid) bond market provides a cheap, efficient way for hedging and speculating on interest rates. For instance, in year 2002, the US treasuries market represented $3 trillion of outstanding debt, with an average daily trading volume of $300 billion (source ISDA).
- Futures: there are various types of Futures, such as
  - Eurodollar Futures: cover short term (between a few days to three months time for the first contract, depending on when the first IMM[3] date occurs) up to medium term contracts, which allows to agree on a future deposit rate.
  - Bond Futures: a Futures contract on a theoretical bond also called *notional bond*.
- Swap: in its vanilla form, this refers to the single-currency market of swaping fixed against floating and vice versa. Swaps are very general structures and can recover many different products. For instance, the

[1] although their contract' s details can in certain case be not that simple.
[2] Like various right to call or put the bond, like bond paying an inflation index or any non standard index.
[3] IMM stands for International Money Market

swap can have amortising notional, as opposed to bullet notional (standard swap), compounding or averaging fixings, can be between different currencies (cross currency swaps), or between two different floating rates (basis swaps). The fixing of the Libor in the swap can be paid immediately (in-arrear swap). The floating rate paid can be a constant maturity swap (CMS, CMT swaps).

- Forward Rate Agreement (FRA): these are forward contracts on a specific rate.
- Asset swap: allows to swap a given bond against the stream of Libor payment plus a spread. It is the spread over Libor that is actually quoted in an asset swap.

Over the last few years, we have seen an important development of electronic trading platforms for these products. Most of these products are nowadays quoted electronically on various computerised exchanges.

OVERVIEW OF INTEREST RATE DERIVATES

The "zoology" of interest rates derivatives has exploded over the last few years as a result of a greater sophistication of the client needs and interest rates hedging technology. Vanilla or very-close-to-vanilla products are:

- *Caps* and *Floors*: which are a strip of simple options (caplets or floorlets) to cap or floor a given interest rate, often a Libor rate.
- *European Swaptions*, which are options to enter in a receiver or payer (depending on the type) swap.

- *Bermudan Swaptions*. These options which gives the right at various exercise date to enter in a payer or receiver swap (depending on the type of swaption) have progressively become more and more vanilla as their traded volume have substantially increased while simultaneously, the pricing technology have become more and more standard. Compared to standard vanilla options, pricing models have to account for the various conditional forward volatility. Although, one factor mean reverting models may make the job through the term structure of mean reversion, two factors models would provide very different hedging strategy. Bermudan swaption can be thought as the most European option plus a switch option to exchange this most expensive European swaption with other possible swaptions.
- Pseudo vanilla swaps, such as
    - *Quanto swap* where the Libor rate is denominated in another currency than the one paid, like for instance a Euro denominated swap on the US Dollar Libor rate. Quanto swap depends on the forward correlation. Innovative way of modelling the dependence between the forward FX and the interest rates includes the use of copula.
    - *CMS, CMT swap,* where the floating rate is a swap rate or treasury rate of constant maturity. It is easy to show that a portfolio of cash settled swaption can statistically replicate a CMS swap. For physically settled swaption, CMS swap can be also replicated by modelling the annuity of the swap as a function of the swap rate. This is equivalent to make a one factor approximation.

- *inverse floater swap*, where one pays a fixed rate minus Libor; often callable.
- *trigger swap*, that pays only if a specific rate hits the barrier, or a reference index, such as an equity stock index or a commodity index, has reached some level (knock-in version). Alternatively, the swap can be cancelled if the reference index hits the knockout level. The risk of this strategy lies in the skew of the smile at the barrier level. Easy modelling of this skew includes shifted lognormal models with stochastic volatility, mapping models between normal and lognormal very equivalent to shifted lognormal models, CEV models with stochastic volatility (for which one can obtain approximation with closed form solution).
- Combination of the above, like triggered quanto CMS swap …etc.

Exotic instruments includes the following:

- Exotic swaps like:
  - *range floater,* which pays if the floating rate is within a range; also referred to as a *range accrual* or *accumulator note*. Exotic range accrual include *callable range accrual* where the option issuer has the right to cancel the structure at certain dates, but also *double-up range accrual* where the issuer can double the size. Other variation includes *triggered range* accrual where the structure is activated if a reference asset has reached some specified level. This third party asset can be commodity or equity, making it a cross asset or hybrid product. This

product includes a smile risk at the barrier in their simple version and in the case of triggered range accrual, an additional correlation risk.

  - *index amortizer swap*, which pays on a notional that amortizes according to the performance of a given rate. This product is much more complex than a swap with a scheduled deterministic amortizing notional as the amortization of the notional in this structure is not known in advance and depends strongly on the full term structure of interest rate and their correlation.
  - *Inflation linked swap*, which pays either the year-on-year inflation, or, in the zero-coupon form, the inflation return from the first date up to maturity.
  - *Parisian barrier type swap*, where in the swap is activated if the reference rate or third party asset stays within a range for a minimum number of days, soft barrier type swap whose barrier is triggered smoothly.

- Exotic caps and floors like:
  - *Asian cap*, which caps the average of the Libor. Asian options are quite popular among investors as they are often cheaper and less sensitive to price manipulation.
  - *CMS cap* and *floor*, where the rate capped (or floored) is a constant maturity swap. CMS derivatives are tailored instrument to trade the steepness or not of the yield curve.
  - *digital cap,* which pays a given fix amount if the rate is above or below a certain level. Digital cap can be considered to be the building block

for many structured products, as any European payoff can be re-examined as a portffolio of digital option or call.

- *ratchet cap,* whose strike resets if the previous Libor fixing has reached some level. The main risk in ratchet cap consists in forward volatility.
- *step up* and *step down caps*, whose strikes can go up and down very similarly to the ratchet cap,
- *chooser cap*, where the option holder has the right to choose a subset of m out of n caplets (m<n),
- *autocap*, also referred to as *non-chooser*, where it is the first m in-the-money caplets which are exercised. Like chooser cap, autocap mainly depends on the term structure of the forward correlation between the different Libors. Part of this correlation term structure is often modelled using mean reversion models. Market models helps greatly to model realistic shape of forward correlation.
- *accrual* also called *accumulator cap*s, whose notional accretes if the Libor or another reference rate stays within a range.
- *knockout cap,* and more generally *barrier caps* whose optionality vanishes (knock-out case) or is activated if the Libor hit the barrier level.
- combination of the above instruments, such as *Quanto knockout cap*, *quanto CMS cap*, *Libor knockin average CMS Cap*, *Equity triggered CMT cap, Equity swap knockout* and so on.

- Exotic swaptions, quantoed or not to eliminate foreign exchange exposure:
  - *knockin, knock out Bermudan* swaptions.

- *volatility bond*, which pays the absolute value of the difference between the fixings of a reference at two consecutive dates, often leveraged.
- *outside barrier swaptions* and *Bermudan swaptions*. Some of these triggered swaptions are using as reference a non-interest rate asset, like a commodity index, or an equity stock or index. In this case, the structure falls into the category of hybrids trades and requires a thorough understanding of correlation risk.

- *Caption* and *floortion*: these products can be seen as compound option since it is an option to enter into a strip of other options. As such, it is often appropriate to have a model accounting for volatility of volatility and realistic forward smile.

2. FIXED INCOME MODELLING AND RISK MANAGEMENT

MODELLING

There exists a wide range of models for interest rate derivatives, out of which the two most popular models are:

- Market models, like BGM in its skew version.
- Hull and White model as it is a highly tractable model, that can be also used for fast pricing.

Just to cite a few, interest rates models can de classified as:

- One factor short rate models like
  - the historical time homogeneous short rate models including the
    - Vasicek model (Gaussian model with mean reverting drift)

$$dr_t = k(\theta - r_t)dt + \sigma dW_t \qquad (1.1),$$

- the Dothan model (lognormal short rate model), that has been, in a sense, the ancestor of the Black Karinski model

$$dr_t = \sigma r_t dW_t \qquad (1.2),$$

- the Cox Ingersoll Ross model (where the short rate follows a square root diffusion), leading to a closed form solution for the discount bond and several options using non-centered chi-square distribution

$$dr_t = K(\theta - r_t)dt + \sigma \sqrt{r_t} dW_t \qquad (1.2),$$

and more generally the model of the CEV type

$$dr_t = K(\theta - r_t)dt + \sigma r_t^{\beta} dW_t \qquad (1.3),$$

- affine term structure models, where the bond rate is expressed as an affine function of the short-term rate:

$$R(t,T) = \alpha(t,T) + \beta(t,T) r_t \qquad (1.4).$$

Affine term-structure models are, in fact, quite general and include as a sub-case the Vasicek and CIR model, as one can show that a necessary and sufficient condition for a model to be affine is that the drift term $\mu_t(r_t)$ and the square of the normal volatility terms $\sigma_t(r_t)$ are affine functions of the short rate. More precisely, if the dynamics of the spot short rate is of the form

$$dr_t = \mu_t(r_t)dt + \sigma_t(r_t)dW_t \qquad (1.5),$$

then any drift and volatility terms of the type $\mu_t(r_t) = a_t + b_t r_t$ and $\sigma_t(r_t) = c_t + d_t r_t$ guarantees an affine term structure model.

- the exponential Vasicek model where the dynamics of the short rate is lognormal and given by

$$dr_t = r_t\left[\theta + \frac{\sigma^2}{2} - a\ln r_t\right]dt + \sigma r_t dW_t \qquad (1.5),$$

- the Hull and White one factor model (also referred by certain authors as the one factor linear Gauss Markov model)

$$dr_t = [\theta(t) - \lambda r_t]dt + \sigma dW_t \qquad (1.6),$$

with $$\theta(t) = \frac{\partial f(0,t)}{\partial T} + \lambda f(0,T) + \frac{\sigma^2}{2\lambda}\left(1 - e^{-2\lambda t}\right) \qquad (1.7),$$

and $f(0,t)$ represents the instantaneous forward rate at time 0 for maturity t

- the Black Derman Toy and Black Karinsky models that are lognormal models. For instance for the Black Karinsky, the dynamic of the short rate is given by:

$$d\ln r_t = [\theta(t) - \lambda \ln r_t]dt + \sigma dW_t \qquad (1.8),$$

- the humped volatility short rate models (Mercurio and Moraleda models (2000)), whose dynamics have been considered to be for the first version:

$$dr_t = [\theta(t) - \beta(t) r_t]dt + \sigma dW_t \qquad (1.9),$$

with $\beta(t) = \lambda - \frac{\gamma}{1+\gamma t}$ (1.10),

while for the second version

$$dx_t = [\theta(t) - \beta(t) x_t]dt + \sigma dW_t \qquad (1.11),$$

with $\beta(t) = \lambda - \frac{\gamma}{1+\gamma t}$ (1.12),

where $r_t = \exp(x_t)$ (1.13),

with x_t being an underlying Gaussian process.

- the Cheyette Beta model, which is a model with a drift, whose evolution is non Markovian.

- the two factor short rates models
  - two factor Gaussian models, where the short rate is the linear sum of two mean reverting Gaussian factors[4]

$$r(t) = x_1(t) + x_2(t) + \varphi(t) \quad (1.13),$$

$$\text{with } dx_i(t) = (\alpha_u - \lambda_i x_i(t))dt + \sigma_i dW_i(t) \quad (1.14),$$

$$\text{with } dW_i(t)dW_j(t) = \rho_{ij}dt \quad (1.15),$$

  - the Hull and White two factor model, model whose mean reversion is itself stochastic

$$dr(t) = [\theta(t) + u(t) - \lambda(t)r(t)]dt + \sigma_1(t)dW_1(t) \quad (1.16),$$

$$\text{with } du(t) = -b(t)u(t)dt + \sigma_2(t)dW_2(t) \quad (1.17),$$

$$\text{with } dW_1(t)dW_2(t) = \rho_{12}dt \quad (1.18),$$

  - the two factor CIR model, the short rate dynamics is the sum of two CIR processes[5]

$$r(t) = x_1(t) + x_2(t) + \varphi(t) \quad (1.13),$$

$$\text{with } dx_i(t) = (\alpha_u - \lambda_i x_i(t))dt + \sigma_i \sqrt{x_i(t)}dW_i(t) \quad (1.14),$$

$$\text{with } dW_i(t)dW_j(t) = \rho_{ij}dt \quad (1.15),$$

- Model specified under the HJM framework by the diffusion of the zero coupon bond dynamics, using either the Ritchken and Sankarasubramanian framework or the Mercurio Moraleda framework.

[4] This current framework can be easily extended to many factors. It is sometimes referred to as equilibrium models when using these mean reverting Gaussian factors calibrated to historical data.
[5] Like the Gaussian n factors models, the CIR models can be extended to multi-factors versions.

- The Libor and swap market model
  - the BGM Jamshidian model or lognormal forward model

$$dL(t,T,T+\delta)=\sigma(T,L(t,T,T+\delta))L(t,T,T+\delta)dW_t^{T+\delta} \qquad (1.16),$$

where $L(t,T,T+\delta)$ is the value at time t of the libor resetting at time T paid at time $T+\delta$, $W_t^{T+\delta}$ is a Brownian motion under the forward measure $Q^{T^{T+\delta}}$.

The relationship between the various measure is the following

$$dW_t^{T+\delta}=dW_t^{T}+\frac{\delta\sigma(T,L(t,T,T+\delta))}{1+\delta L(t,T,T+\delta)} \qquad (1.17)$$

  - the market model with CEV skewed volatility

Similar to the previous with the exception that the diffusion is following a CEV process

$$dL(t,T,T+\delta)=\sigma(T,L(t,T,T+\delta))L^{\beta}(t,T,T+\delta)dW_t^{T+\delta} \qquad (1.18),$$

  - the market model with jumps

Similar to the normal BGM model with a jump in the diffusion of the forward Libors.

$$dL(t,T,T+\delta)=\sigma(T,L(t,T,T+\delta))L(t,T,T+\delta)dW_t^{T+\delta}+\lambda dN_t \qquad (1.19),$$

  - the market model with stochastic volatility

Similar to the normal BGM model with a stochastic volatility in the diffusion of the forward Libors.

$$dL(t,T,T+\delta)=\sigma(T,L(t,T,T+\delta))L(t,T,T+\delta)dW_t^{T+\delta}+\lambda dN_t \qquad (1.20),$$

with $d\sigma(T,L(t,T,T+\delta))=\alpha_t dt+\beta_t dW_t^{T+\delta}$ (1.21),

A very important distinction in the universe of interest models is based on whether the given model tries to reproduce the observed yield term structure (that is, the yield of zero-coupon bonds for various maturities, as observed today in the market), or takes this term-structure as input. For example, a 1-factor Hull&White (1993, 1994) model is based on the dynamics of a short rate and it is through the inclusion of a time-dependent function in the drift term of the short rate dynamics that one can fit the observed yield curve. Another starting point, which by construction would reproduce the observed yield curve, would be to model the evolution of the entire term structure (the forward rates): this is the case of the *Heath Jarrow Morton* (HJM, 2000) family of models, where it turns out that one needs to specify the initial structure of all forward rate volatilities. Also, and very importantly, enforcement of the no-arbitrage condition in the economy leads to the condition that the drift of the forward depend on the past history of the volatility of the forward rate! The drawback with the HJM approach is that the resulting short-rate process is generally non-Markovian (so when implementing it in practice, one would end up with non-recombining trees which means that the number of nodes will grow exponentially with the number of steps) and can only be turned into a Markovian process for suitable specification of the forward-rate volatilities.

Another important issue with quite a number of *short-rate* models is how to calibrate the parameters of the model to the market prices of liquid instruments such as caplets and swaptions. One can see this as a problem of minimising the global mismatch between model-derived prices of these instruments and the corresponding market-observed prices. The global-mismatch function viewed formally, can represent a pretty “tough” (multi-valley, e.g.) multi-dimensional surface on which one is asked to carry out the minimization. This, in turn calls for the use of sophisticated global constrained optimization methods, where the constraints come from various considerations: if correlation and volatility are in the list of parameters of the to-be-calibrated model, then the correlation should always be non-greater in absolute value than unity, and volatility should be a non-negative number. Any calibration of the model to market prices should address these constraints. In view of the possible multi-valleyness of the surface on which we carry out the optimization, one should try different starting points (*multi-start* methods). Of course, if no important market movement has occurred between two calibration dates, then one can use, to accelerate the search, as starting point the parameter values obtained at the first of these two dates.

We will, now focus more on two families of interest rate models that have gained increasing interest amongst academics and practitioners: the Libor Market Models and the Markov Functional Models.

- *Forward Libor* and *swap market model*: contrary to short-rate models, these models try to model the dynamics of a market-observable rate: the forward Libor, a discretely compounded rate, and the swap rate. This very fact renders them more appealing than their short rate counterparts. Focussing on the *forward Libor market* model (FLM, in the sequel), we will note that this model provides theoretical ground for the, otherwise self-contradictory, way market practitioners use to price caplets: they use the Black&Scholes formula, which formally tantamounts to treating the discount factor, used to discount the payoff at expiry, as non-stochastic (which is equivalent to taking the interest rate as non-stochastic) but taking the interest rate in the payoff to be stochastic and more precisely assume it to be lognormally distributed. FLM models prescribe, at this point, that the above two-step procedure is legitimate solely under the measure whereby the numeraire is the discount bond (zero-coupon) corresponding to the expiry of the caplet (a caplet covering the period $[S,T]$ paying at T, matures at S and expires at T). But then, forward rates with different maturities will each one be lognormally distributed under the measure corresponding to the expiry of the given rate. What, then, one needs to do, in a pricing context, is to apply a measure transformation to express the dynamics of all forward rates under the same measure (this could even be the risk-neutral measure).

What FLM does for caplets, forward swap rate models (FSM) do for swaptions. The latter models describe the evolution of forward swap rates assuming that a given forward swap rate is lognormally distributed under the measure corresponding to the numeraire being the PVBP (present value of a basis point). This is the denominator in the expression for the forward-starting-swap rate, $y(t,T_1,T_n)$, for a swap starting at T_n and having its last payment at time T_N: $y(t,T_1,T_n)=\dfrac{DF(t,T_n)-DF(t,T_N)}{\sum_{i=n+1}^{N}DF(t,T_i)(T_i-T_{i-1})}$

However, forward swap and Libor rates are taken to be lognormally distributed under different measures! If one, then, takes into account that a swap rate is a superposition of Libor rates, then one sees that the two cannot be lognormally distributed at the same time. Thus, it is not legitimate to use both market models at the same time. A way to go around this problem is to choose on of the two models, say the FLM so that caplets prices are recovered immediately, and perform Monte-Carlo simulation on the other reference instruments (the swaptions, of FLM was chosen). In order to make up one's mind as to which model to pick up, the following can be done: if using FLM, calculate the total mismatch between model-implied and market-observed swaption prices. If using the SLM, then do the same with caplet prices. The model that gives the smallest mismatch is to be retained. It is important to note at this stage that whereas short-rate models are based on the dynamics of a low dimensional Markov process (the rate $r(t)$ itself), market models use a much higher-dimension Markov process: this renders these latter models less tractable in general.

Another family of models comes to fill in the gap:

- *Markov Functional models (MF):* initially proposed by Hunt, Kennedy and Pelsser (1998), these borrow the idea of low-dimensional Markov process, denote it by x(t), describing the economy, from short-rate models. Once defined this process, one then needs to stipulate the way it relates to assets in the economy, if one is interested in recovering the correct distribution of relevant market rates (a short digression is in order to note that when it comes to modelling and pricing, especially of exotic products, one is concerned about both fitting the marginal distribution of interest rates, as these are deduced by market-observed option prices, and the joint distribution of these rates). MF's go one step further in "stealing ideas" from the short-rate models: in the latter ones, the spot rate uniquely determines the values of a (arbitrary) discount bond. MF's will stipulate that the same holds true for the relation between $x(t)$ and the discount bonds.

More formally, a MF is built upon the following assumptions, where (N,Q) is the numeraire pair, that is, the numeraire and the corresponding measure:

- $x(t), t \in [0,T]$ is Markovian under Q
- zero coupon bonds are functionals of the above process:

$$DF(t,u) = DF(t,u,x(t)), \quad u > t$$

where $0 \le t \le \vartheta_T \le T$, ϑ_T being the *boundary-curve,* a function of maturities (if T^* is a terminal maturity of interest to the given product to price, then, $\vartheta_T = T$ if $T \le T^*$, and $\vartheta_T = T^*$ if $T \ge T^*$).

- The numeraire $N(t)$ itself is a functional of $x(t)$: $N(t,x(t))$.

As a next step, to completely specify the model one needs to:

- Specify the law of $x(t)$ in the given measure
- Deduce the functional form of the pure discount bonds on the aforementioned boundary
- The functional form $N(t,x(t))$

Thus, one has the freedom to “play around” with the functional forms of the pure discount bonds and the numeraire itself and can therefore meaningfully use this double freedom, first to calibrate the model, by an appropriate choice of the functional dependence of discount bonds, to the prices of pertinent market instruments and secondly, by an appropriate choice of the numeraire, recover the marginal distributions of pertinent market rates (say swap rates).

To epitomise, when it comes to building a model for interest rates, one should pay attention to the following points:

- The model should be arbitrage-free.
- Model-implied prices for liquid instruments should match as much as possible the market-observed prices for these instruments.
- The model should be robust (without making any "compromises" on the "connection-to-reality" front) and as easy as possible to implement.

RISK MANAGEMENT

In order to hedge fixed income instrument, traders and risk managers use various risk concepts such as:

- *Duration* and *convexity* of instruments which consists in assuming mainly a flat term interest rates term structure. This is a primary tool for bond trading.
- The interest rate *delta* (and *gamma*), which admits several definitions: parallel shift of the interest rate curve, bump of the various instruments used to build the curve, bucket delta to calculate the sensitivity of the portfolio at various portions of the curve, factor risk and PCA (Principal Component Analysis) risk which consists in computing the risk to some factors.
- *Vega*, which also admits many definitions but is commonly defined as the sensitivity with respect to the volatility of the instruments used to calibrate the interest rates term structure models. Traditionally, one tries to see if the product can be seen as a swaption or a cap product and to use only

the volatility of one type of instrument. The diagonal of the swaptions' matrix is often used as a starting point for the calibration of the interest rates model. With Libor market model, the calibration is global in the sense that almost all the market instruments are used.

- *Scenario analysis*, *value-at-risk* (both linear and non-linear), extreme VAR and stress testing, where in addition to the curve and volatility, the other market risk factors like correlation, mean reversion and funding are used for various risk and sensitivity scenarios.

3. FIXED INCOME MARKET EVOLUTION

Like in other markets, exotic products become more and more standard and vanilla in the interest rate universe. The flow business is very different in spirit from the exotic one. The flow business consists in developing a franchise, in the sense that investment banks offer to clients adequate technologies and tools (like electronic trading platforms, web interfaces, e-commerce marketing) to fulfil customers flows, making profits on proprietary overlay, gathering valuable knowledge of market information and better understanding of activities and needs of key market participants.

As the business becomes more and more vanilla, margins erode while competition for order flow increases. Appropriate technological progress helps to downsize costs.

Entry category: markets and instruments.

Scope: markets and instruments; structures; trading processes, emphasis on risk management, financial engineering issues .

Related articles: Bond types; Hybrids; Commodity markets; Equity markets (overview); Money markets (overview); Foreign exchange.

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References

Brigo Damiano, Mercurio Fabio (2001*) Interest Rate Models. Theory and Practice,* Springer Finance.

Heath, D., Jarrow, R., Morton, A. (1992*) Bond pricing and the Term Structure of Interest Rates: A new methodology.* Econometrica 60, pp. 77 -105.

Hull, J., White, A. (1993*) One-Factor Interest Rate Models and the Valuation of Interest rate Derivative Securities.* Journal of Financial and Quantitative Analysis 28, pp. 235 - 254.

Hull, J., White, A. (1994*) Numerical Procedures for Implementing Term Structure Models I: Single-Factor Models.* The Journal of Derivatives 2, pp. 7 - 16.

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The views and opinions expressed herein are the ones of the author's and do not necessarily reflect those of Goldman Sachs or Deutsche Bank.

Hunt, P., Kennedy, J. , Pelsser, A. (1998), *Markov-Functional Interest Rate Models.* Working Paper, University of Warwick.

Hunt, P., Kennedy, J. (2000*) Financial Derivatives in Theory and Practice.* Wiley, Chichester.

Pelsser, Antoon (2000*) Efficient methods for Valuing Interest Rate Derivatives,* Springer Finance.