Equity forward contract

INTRODUCTION

An equity forward contract is an agreement between two counterparties to buy a specific number of an agreed equity stock, stock index or basket at a given price (called strike price) at a given date. Like for any forward contract (See forward contracts), no cash changes hands until the maturity of the contract. Equity forward contracts are cash settled in most cases. At maturity, the two counterparties exchange a cashflow equivalent to the difference between the stock closing price and the strike price.

VALUATION

Standard valuation is to use the forward neutral measure. Denoting by T the expiration date of the forward contract, by $B(0,T)$ the price of a zero coupon paying 1 at time T, by K the strike of the forward contract and by S_T the forward underlying, the equity forward contract is worth

$$P = B(0,T)E^{Q_T}\left[(S_T - K)\right] \quad (1.1),$$

where Q^T is the T forward risk neutral measure. The strike for which the forward contract is worth zero is called the forward price of the forward underlying and is given by

$$Fwd(T) = E^{Q_T}\left[S_T\right] \quad (1.2),$$

which under the assumption of log normal return with countinuous dividend yield q_s and risk free rate r_s is given by

$$Fwd(T) = S_0 \exp\left(\int_0^T (r_s - q_s) ds \right) \qquad (1.3).$$

Equity forward contracts exist also in the form of contract for differences. In this particular case, the two parties agree to exchange at the maturity of the contract, a cashflow equivalent to the difference between the opening and closing prices multiplied by the number of shares detailed in the contract. CFDs are traded on margin, do not incur stamp duty.

Many power derivatives houses trading big volume in equity derivatives hedge their risk with equity forward contacts. As opposed to equity futures, these over the counter contracts allows to hedge risk against specific stocks, funds, basket of stocks, funds as opposed to equity futures that concerns major equity stock indices like for instance the S&P 500 futures.

In contrast to equity index futures, tailor made equity forward contract bear no basis risk as the underlying is exactly the one of the equity derivatives positions

DETAILS OF EQUITY FORWARD CONTRACT

The full specification of the equity forward contracts is the following:

- name(s) of the underlying(s): this can be a stock, a stock index, a fund, a basket of stocks, stock indexes or funds

- strike price: the type of strike can be absolute meaning , relative, meaning that it is a percentage of a specified spot close,
- number of share per contract: often a 100.
- expiration date: the date used to close the contract.
- settlement date: often the expiration date plus two-business day.
- denomination of the contract: in which currency the contract will be settled.

QUANTO

Quanto equity forward contracts are contract paid in another currency than the one of the underlying equity. The quanto term can be interpreted as a cross gamma effect between the forward FX rate and the forward stock. Assume that we want to value a forward contract denominated in currency X that pays at time T the price of a stock S_T normally denominated in currency Y. This pricing boils down to value S_T: under the forward X risk neutral measure:

$$E^X[S_T] \tag{1.4}$$

Let us denote by $B^X(t,T)$ (respectively $B^Y(t,T)$) the price at time t of a X (respectively Y) zero coupon paying 1 X (respectively Y) at time T and by $U^{Y/X}(t)$ the spot exchange rate at time t, number of unit Y of for one unit of X. Like for any change of measure, The Radon Nikodym derivative for the change measure from X forward neutral to Y forward neutral measure is given by the ratio of the numeraires at time T divided by the same ratio at time 0:

$$\frac{dQ^X}{dQ^Y} = \frac{B^X(T,T)U^{Y/X}(T)}{B^Y(T,T)} * \frac{B^Y(0,T)}{B^X(0,T)U^{Y/X}(0)} \tag{1.5}$$

Introducing the forward FX $U^{fwd\,Y/X}(t) = \frac{U^{Y/X}(t)B^{X}(t,T)}{B^{Y}(t,T)}$ and the forward stock $S_t^{fwd} = S_t \exp\left(\int_t^T (r_s^{Y} - q_s^{Y})ds\right)$, the expression (1.4) becomes:

$$E^{X}[S_T] = E^{Y}\left[S^{fwd}{}_T U^{fwd\,Y/X}(T)\right] * \frac{1}{U^{fwd\,Y/X}(0)} \qquad (1.6).$$

Assuming that the forward FX $U^{fwd\,Y/X}(t)$ and the forward stock S_T^{fwd} are lognormally distributed with respective lognormal volatility $\sigma^{U^{fwd\,Y/X}}(s)$ and $\sigma^{S^{fwd}}(s)$ and with a time dependent correlation of ρ_s, expression (1.6) transforms into:

$$E^{X}[S_T] = S_0^{fwd} \exp\left(\int_0^T \rho_s \sigma^{S^{fwd}}(s)\sigma^{U^{fwd\,Y/X}}(s)ds\right) \qquad (1.7).$$

The exponential terms is referred to as the quanto correction. Quanto correction can be quite susbstantiantial for equity forward of medium to long term maturity (more than 3 years). The quanto measure using correlation is quite standard in trading institutions. Howver, one of the problem of using lognormal correlation is the difficulty to estimate accurately the correlation coefficient. Partial solutions (in decreasing order of use and practicability) are to use:

- covariance (as a less volatile measure of the dependence between the forward Fx and the forward stock). In addition, one can show that one can lock up covariance when delta and gamma hedging
- cointegration between the forward FX and the forward stock.

- copula as a way of capturing a desired dependence between the two variables.

DIVIDEND

Another important point for evaluating equity forward contract is to use accurate representation of dividends. Although a continuous yield dividend has the great advantage to be relatively simple, this concept can be quite unrealistic for single stock equity forward. Usually one prefers to use lump sum dividends and compute an equivalent term structure of continuous yield dividend, with an additional variable to weight the dividend in case of a forward expiring on a dividend date.

Dividend forecasts are usually quite reliable and can be use with confidence. More advanced modelling of dividends includes stochastic dividends but is not very used in practice.

LIQUDITY RISK AND TRADING OF EQUITY FUND FORWARD

For very illiquid stock, equity forward price has to include a liquidity risk measure. Moreover for equity forward on funds, trading desks often agree bilaterally with fund institutions on certain trading limit and liquidity in order to hedge their position. Since funds have only one quoted daily price, it is also important for trading desk to regroup their order in one go before a cut-off time to be determined with the trading institution.

Last but not least, trading desk often use a proxy based on the index tracked by the fund to compute an estimate of the fund close and to use this forecast for daily trading and hedging.

Eric Benhamou[1]

Swaps Strategy, London, FICC,

Goldman Sachs International

Entry category: forwards.

Key words: forward futures market. Basket forward.

Related articles: forward contracts, contracts for difference.

[1] The views and opinions expressed herein are the ones of the author's and do not necessarily reflect those of Goldman Sachs