

Bond futures

INTRODUCTION

Bond futures contracts are futures contracts that allow investor to buy in the future a theoretical government notional bond at a given price at a specific date in a given quantity. Compared to other futures, bond futures are slightly more complicated as the underlying bond of the futures contract is not a physical bond but rather a theoretical notional bond determined by the basket of available deliverable government bonds issued in the market. Bond futures are very liquid futures contracts and among the most traded futures contracts.

The most common ones are:

- In the US markets: US Treasury bond Futures, often referred to as T Bond Future.
- In Europe: Bund Future (Germany, Euro denominated), Gilt Future (UK, British Pound denominated), Notional contract (France, Euro denominated).

Bond futures are widely used to hedge interest rate risk on long maturities, especially by swap dealers that needs to cover their risk against various points of the interest rate curve. Bond futures bear an additional risk often referred to as the basis risk compared to swaps. Before reviewing the various concepts of bond futures and its complicated delivery mechanism, let us give practical details for most common contracts.

CONTRACT DETAILS

Table 1 gives the details of the most common bond futures: T-Bond, the Gild Future, the Bund Future¹ and the French notional contract.

Contract	US T-Bond	Bund	UK Gilt	Fr Notional
Delivery months	March (H), June (M), September (U), December (Z)	March (H), June (M), September (U), December (Z)	March (H), June (M), September (U), December (Z)	March (H), June (M), September (U), December (Z)
Quotation	Percentage	Percentage	Percentage	Percentage
Contract Size	\$100,000	€100,000	£100,000	€100,000
Coupon	8%	6%	7%	3.5%
Tick Size	1/32 = \$31.25	0.01 = €10	0.01 = £10	0.01 = €10
Last Trading day	7 working days prior to the last business day in expiry month	2 business day prior to the delivery day	2 business day prior to the delivery day	2 business day prior to the delivery day
Delivery day	Any business day in delivery months (seller choice)	10th calendar day of the delivery month or the next following business day	Any business day in delivery months (seller choice)	3 rd Wednesday of the delivery month
Settlement	Any US Treasury bond with more than 15 years T Bond non callable for 15 years	Any Bund with maturity between 8.5 to 10.5 years Min. outstanding amount of €4 billion	Any deliverable Gilt with maturity between 8.75 and 13 years	Any bond with maturity between 8.5 to 10.5 years Min. outstanding amount of €6 billion
Margin requirement	\$5000 initial \$4000 maintenance	€2000 initial	£2000 initial spread margin £250	

Table 1: characteristics of common bond futures

¹, The Bund futures is now the most liquid bond future world-wide

INVOICE PRICE AND CONVERSION FACTOR

Government bond futures are based on a notional bond, which is a theoretical bond whose price is inferred from market physically available bonds. The potentially deliverable bonds need to satisfy certain criteria (see table 1). Like for any other bond, the invoice price of the bond future has to account for the accrued interest on the delivered bond.

$$\text{Invoice price} = \text{Invoice Principal Amount} + \text{Accrued interest} \quad (1.1)$$

The invoice principal amount received by the short has to account for the characteristic of the bond delivered. To adjust for these characteristics, the short receives a total notional amount equal to the standard notional times the **conversion factor (CF)**. The idea behind the conversion factor is to make the various bonds equal provided that the interest rate curve was flat with a given yield² (see conversion factor)

$$\text{Invoice principal amount} = \text{Future price} * \text{Future Notional} * \text{CF} \quad (1.2)$$

² The conversion factor is computing the value of the bond for a flat yield. In the case of the Bund futures contract, the conversion factor assumes a 6% yield while it is 8% for the T-Bond futures. For a bond with n coupon: $(C_i)_{i=1..n}$ paid at time $(T_i)_{i=1..n}$, this leads to

$$CF = \sum_{i=1}^n \frac{C_i}{(1 + 6\%)^{T_i}} + \frac{100}{(1 + 6\%)^{T_n}}$$

On can notice that if the coupons are higher than 6%, the conversion factor should be greater than 1 and the opposite is also true.

CHEAPEST TO DELIVER

In case of delivery, the short decides which bonds to deliver among the list of potential bonds. Obviously, she chooses the **cheapest-to-deliver**, referred to as the CTD.

When delivering the bond i :

- She receives the quoted futures price F_T times the conversion factor CF^i plus the accrued interest AI_T^i : $F_T * CF^i + AI_T^i$
- She pays a total cost of delivering the bond given by the dirty price of the bond, that is equal to the clean price B_T^i plus the accrued interest. AI_T^i

The cheapest to deliver bond is therefore the one that maximises her profit:

$$i_0 = \text{ArgMax}_i (F_T CF^i - B_T^i)$$

Example: using the example of the table 2 and that the Bund future price is 109.2, we get that the bond would cost respectively: 1.69 (=105.75-0.9529*109.2), 2.38, 3.11. The cheapest to deliver would therefore be the first bond.

Many parameters influence the cheapest to deliver bond. When yields are higher than the assumed yield for the conversion factor (6% for the Bund futures and 8% for the T-Bond future) the conversion methodology tends to favour low coupon and long maturity bonds. Similarly, when yields are lower, cheapest to deliver bonds are often high coupons, low maturities. Also when

the yield curve is upward sloping (respectively downward sloping), long-maturities (respectively short-maturities) bond are preferred. Lastly, some bonds trade at a premium because of particularities that makes them attractive like bonds whose coupon can be stripped easily from the bond.

MODELLING OF THE SHORT'S OPTIONS

More accurate modelling of the delivery of the bond is quite complicated. There are a variety of embedded options that the short party holds. First, the short is long an option to choose any bond within the deliverable bond basket, referred to a **switch or quality option**. The short may hedge the bond futures using an assumed cheapest to deliver (with its forward price computed accurately) and a switch option to change from the assumed cheapest to deliver to another bond. So even if she holds a specific bond against delivery, if the curve shifts, she may decide to deliver another cheaper bond. There is also a **timing option** when the bond hold for delivery pays a coupon higher than the cost of financing of the spot position (the repo rate). If the bond provides a coupon higher than the repo rate implied to calculate the forward value of this bond, the short would be better off delivering the bond as late as possible. When one uses Black Scholes to value these small details, one finds that this is very negligible at first sight and can be ignored in most cases.

WILD CARD PLAY AND END OF THE MONTH OPTION

Two embedded options hold by the short have received more attention, as there are the most important options. The wild card play refers to the US T=

Bond futures. There is a time delay of roughly 2-hour between the close of the CBOT Treasury bond futures market (at 2pm Chicago time) and the trading of Treasury bond market (at 4pm Chicago time). Furthermore, the short can wait until 8 pm to issue to the clearinghouse a notice of intention to deliver. This gives the short the optionality, referred to the **wild card play**, to choose the best timing to buy the bond at the cheapest price. Lastly the end of the month option refers to the fact that the short can wait until the last day of the trading month to decide whether to deliver or not while the last trading day is 7 business day prior to the end of the month for the T-bond and 2 business day for the Bund futures contract.

Coupon Rate (%)	Maturity	Conversion Factor	Quote Price	Cost
5.25	04.07.10	0.9529	105.75	1.69
5.00	04.07.11	0.9312	104.07	2.38
5.00	04.07.12	0.9256	104.19	3.11

Table 2: Example of deliverable bonds.

Price of the Bund futures 107.73

Entry category: futures

Scope: contract details, notional bond, Cheapest to deliver, delivery option, uses, cost of carry, model for bond futures, repo.

Related articles: repurchase agreement, bond valuation.

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³ The views and opinions expressed herein are the ones of the author's and do not necessarily reflect those of Goldman Sachs

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